Online Algorithms with Lookaround

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Distributed Algorithms Group
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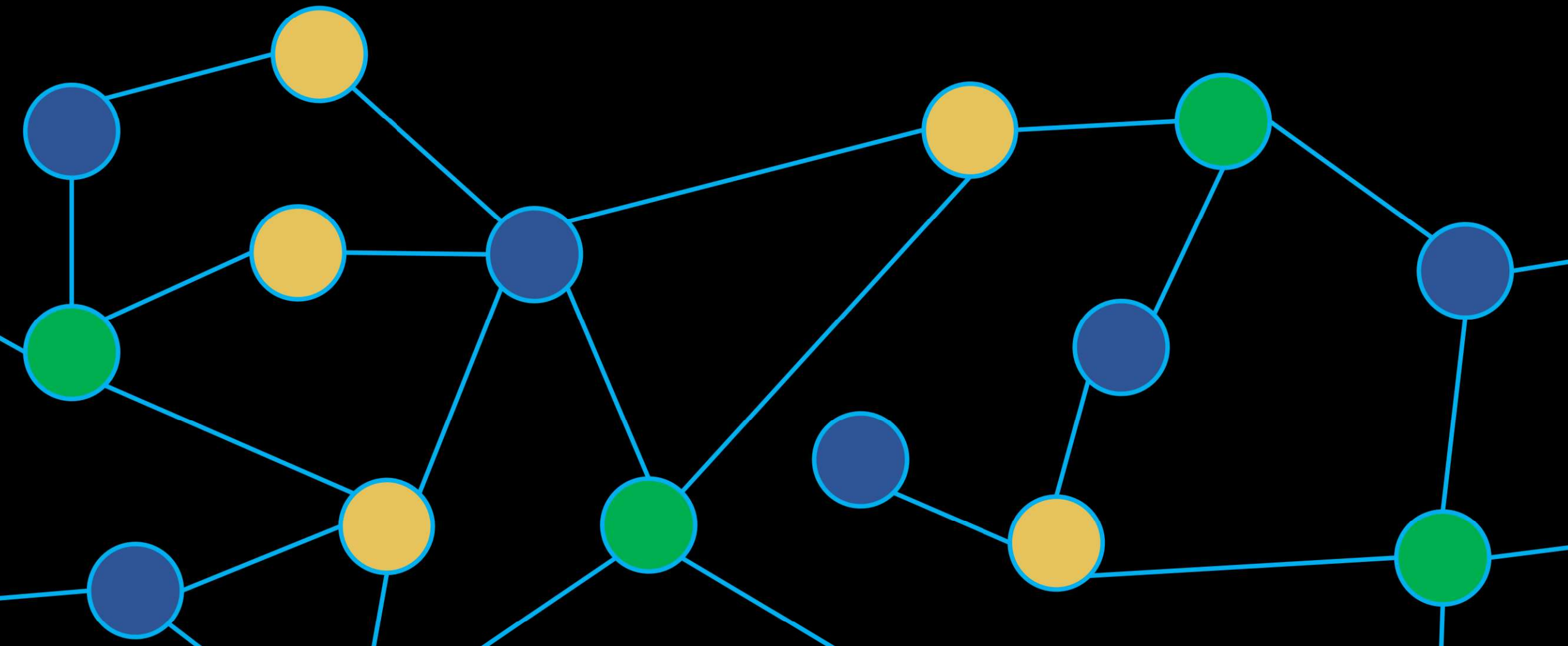
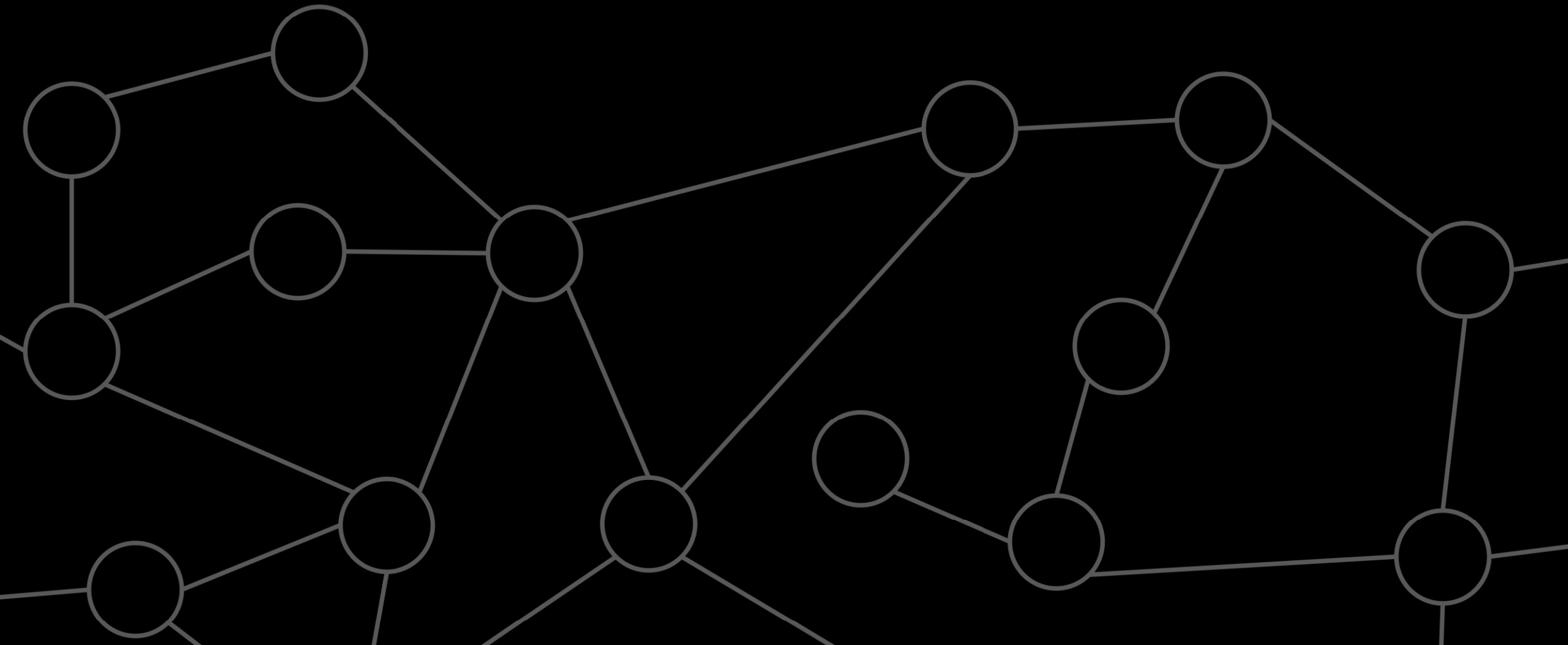
Joint work with Amirreza Akbari, Henrik Lievonen, Joona Särkijärvi, Jukka Suomela

Graph Coloring



Online Algorithm



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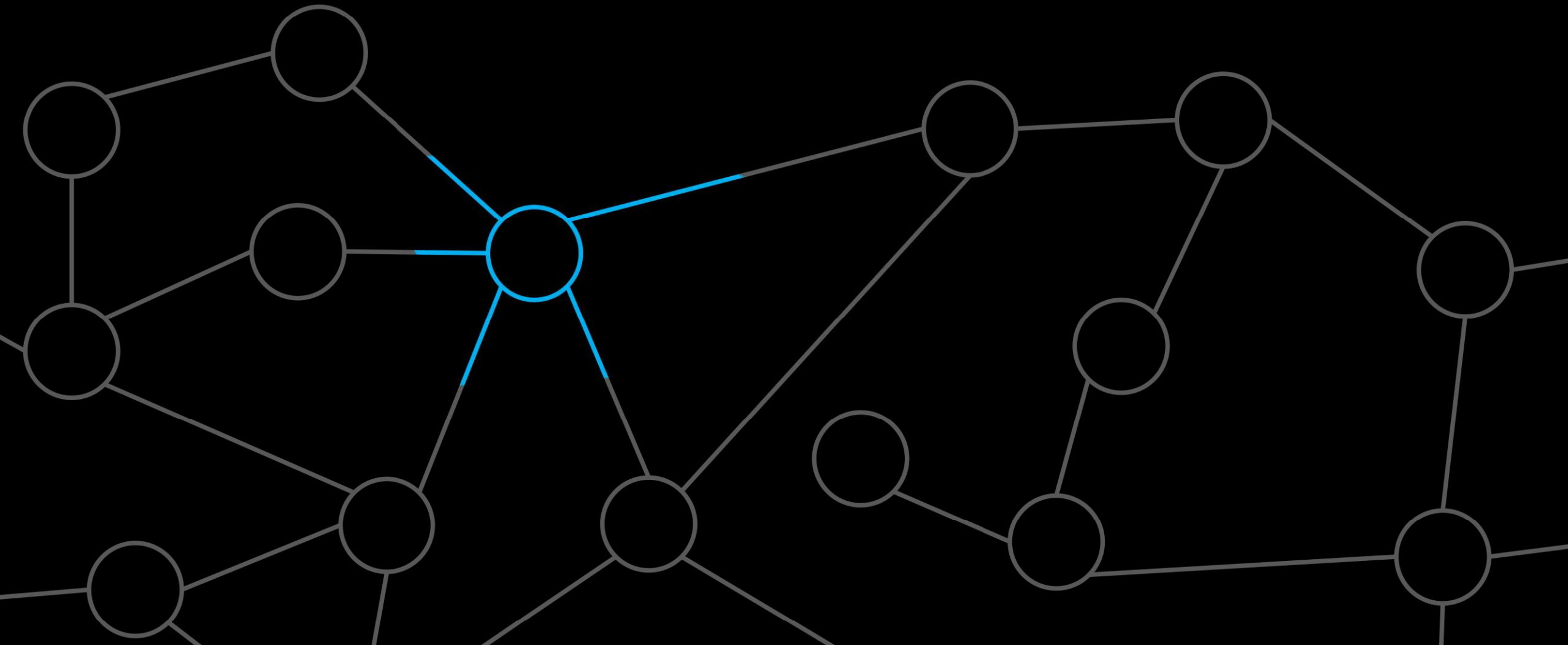

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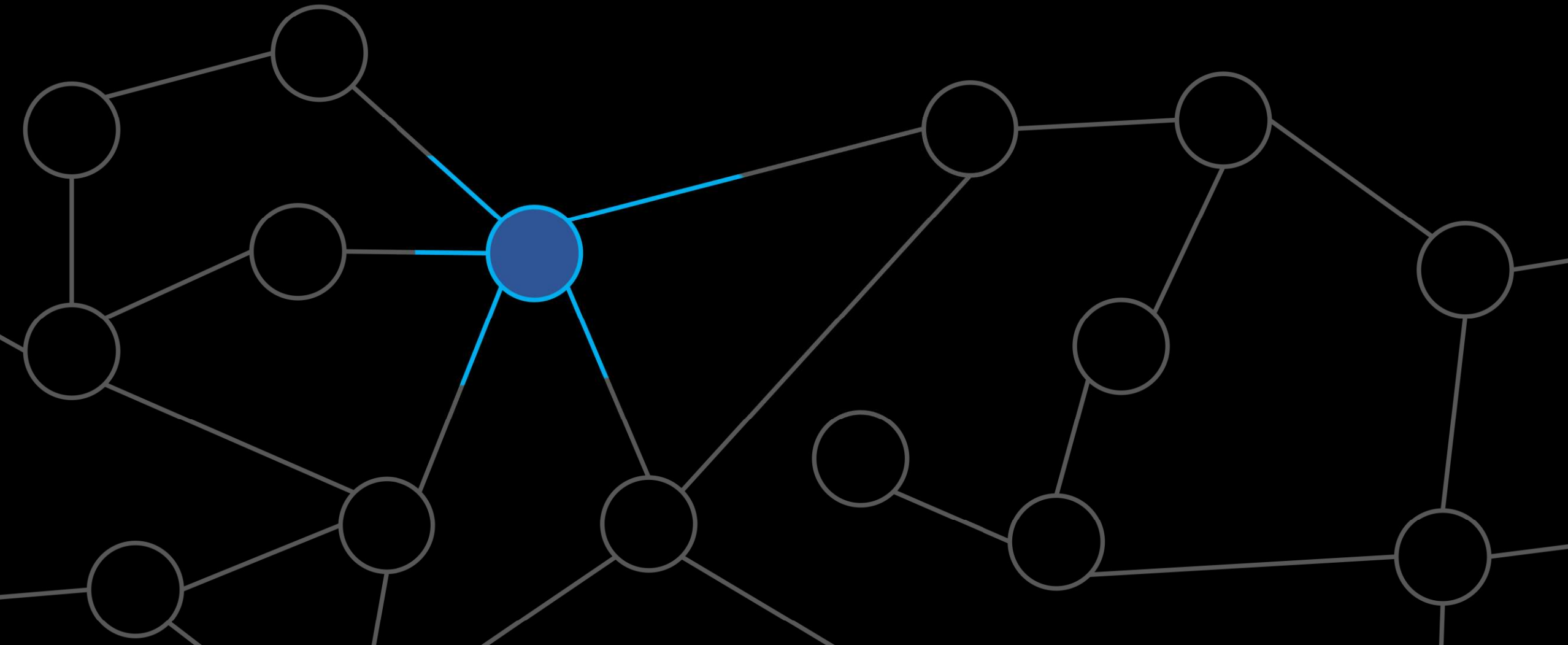
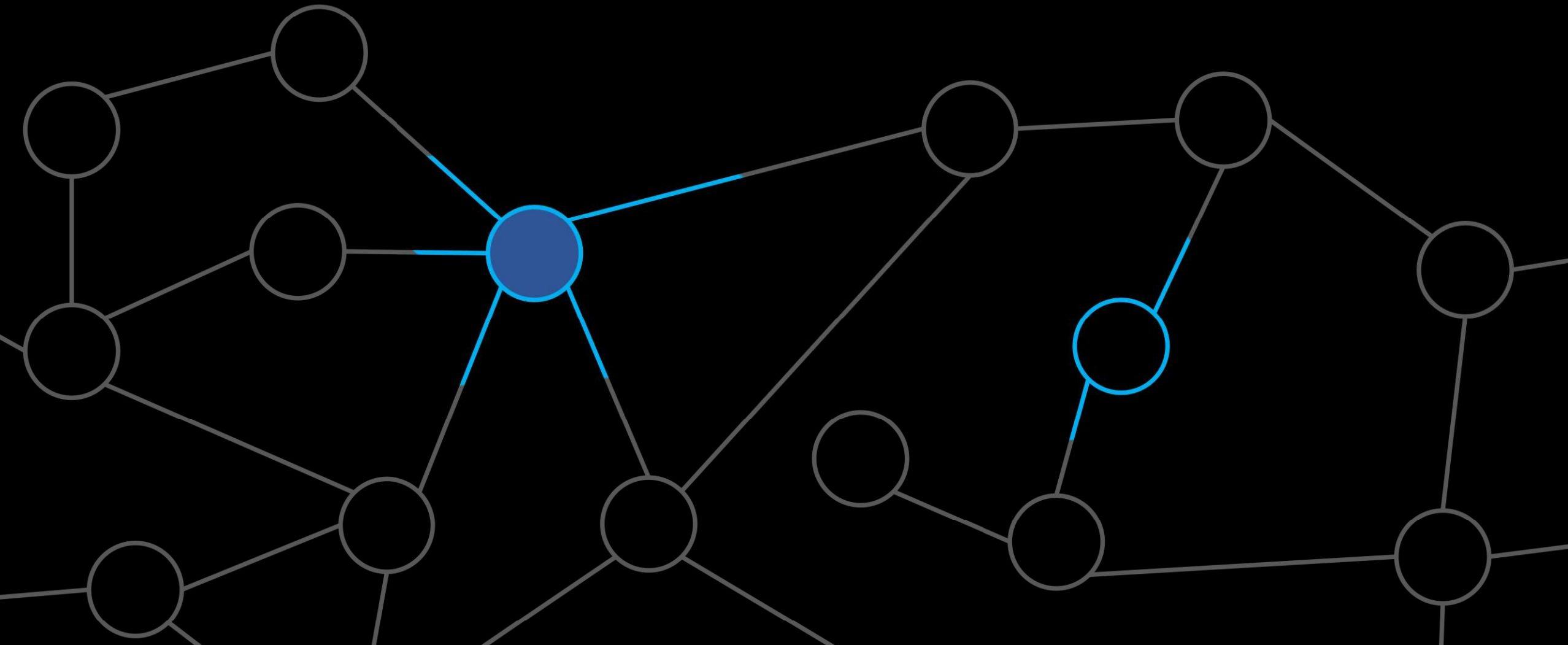
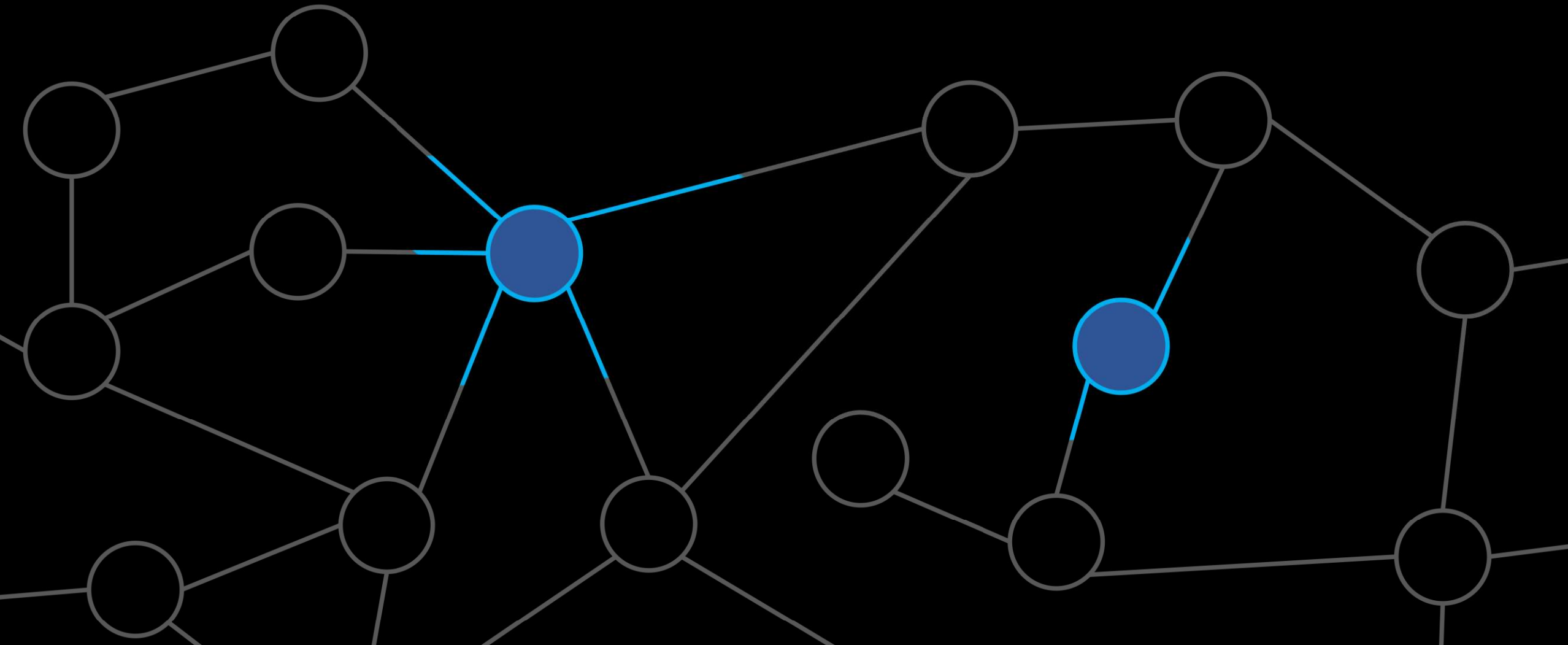


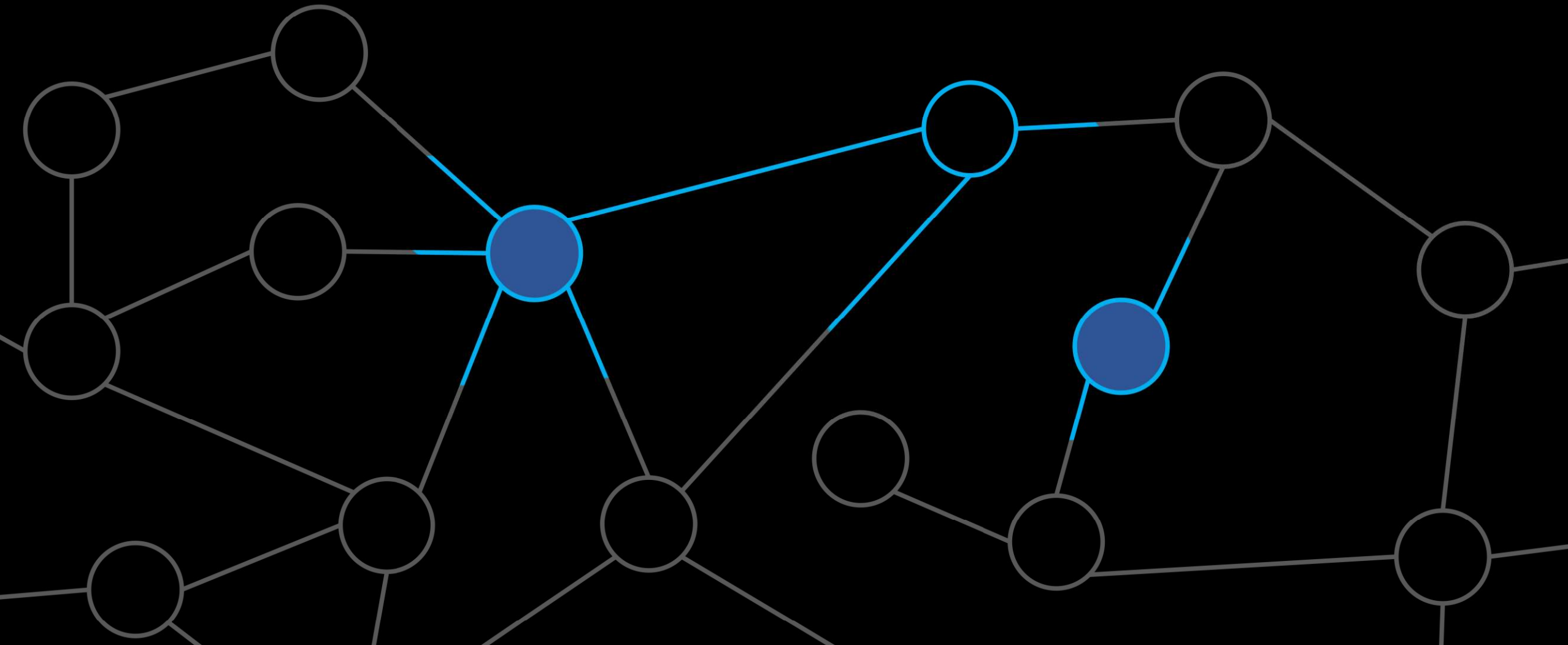
Online Algorithm



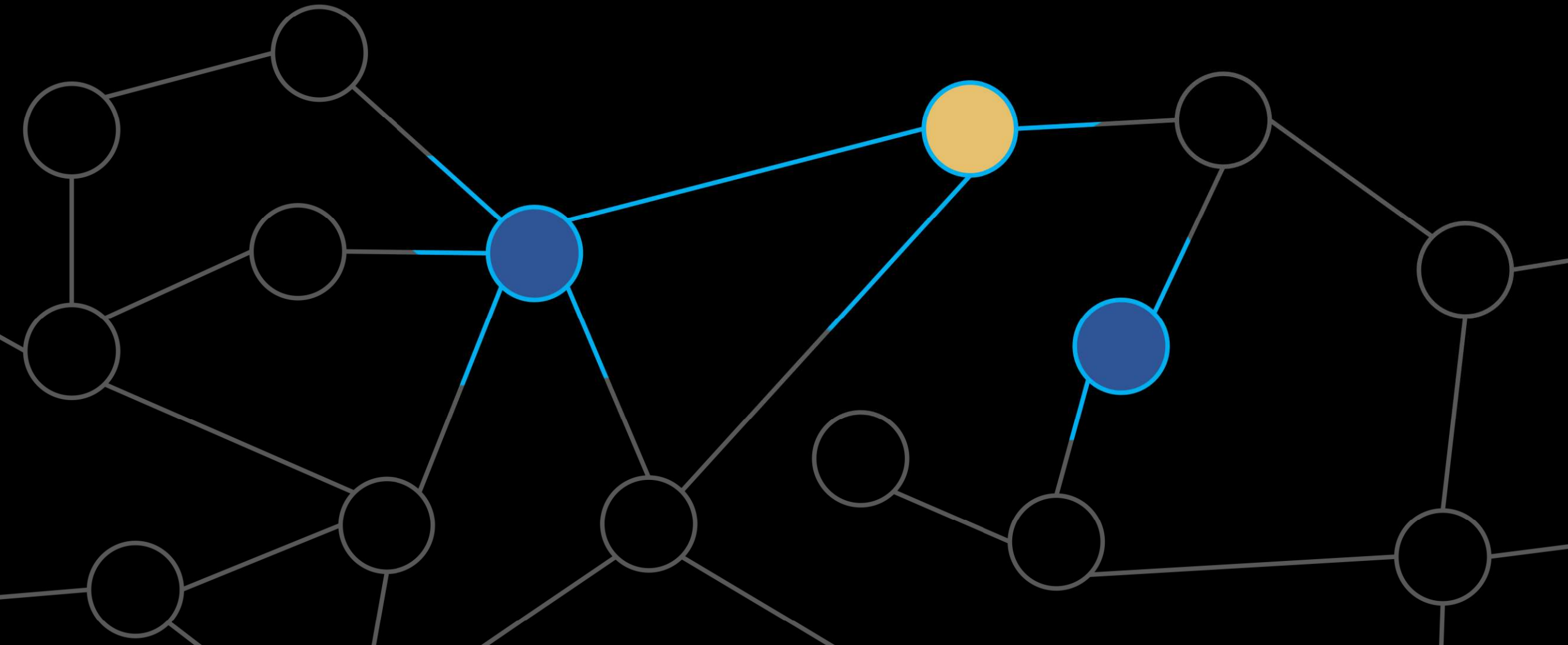
Online Algorithm



Online Algorithm



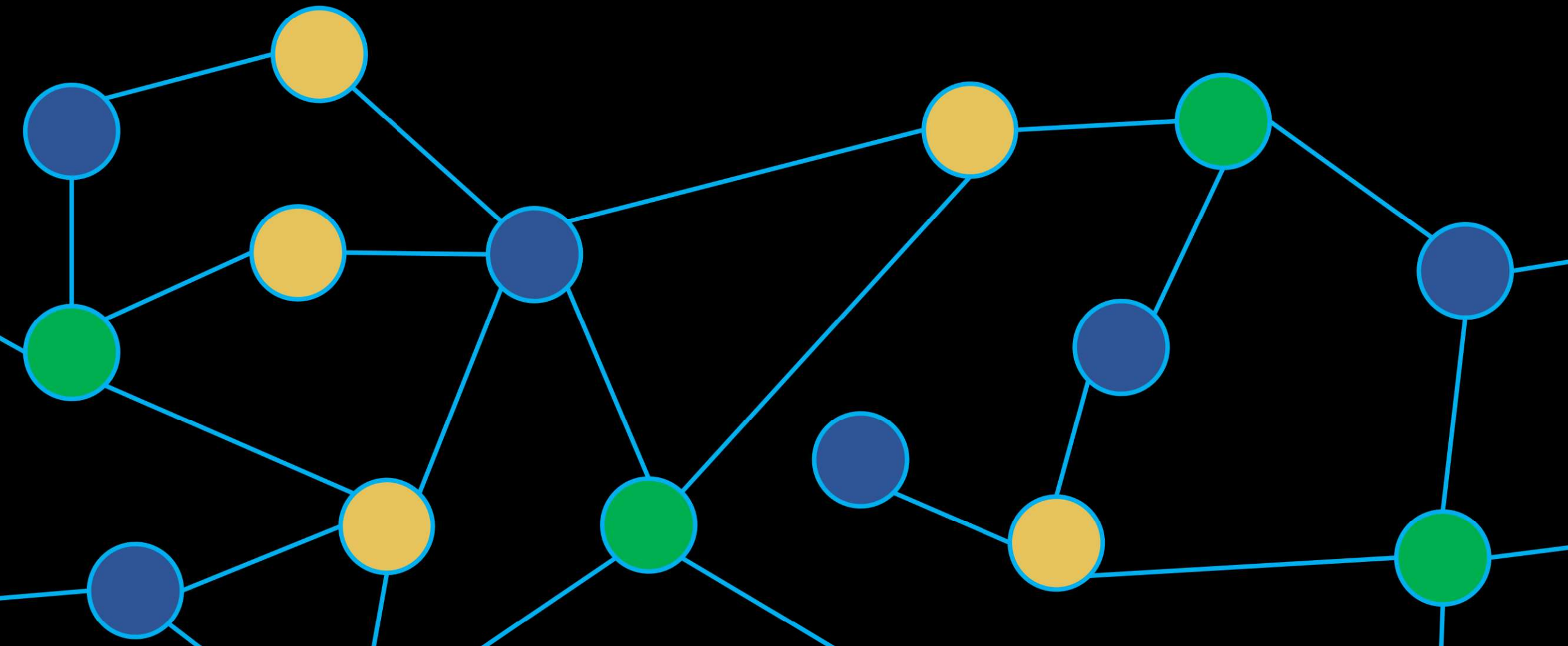
Online Algorithm



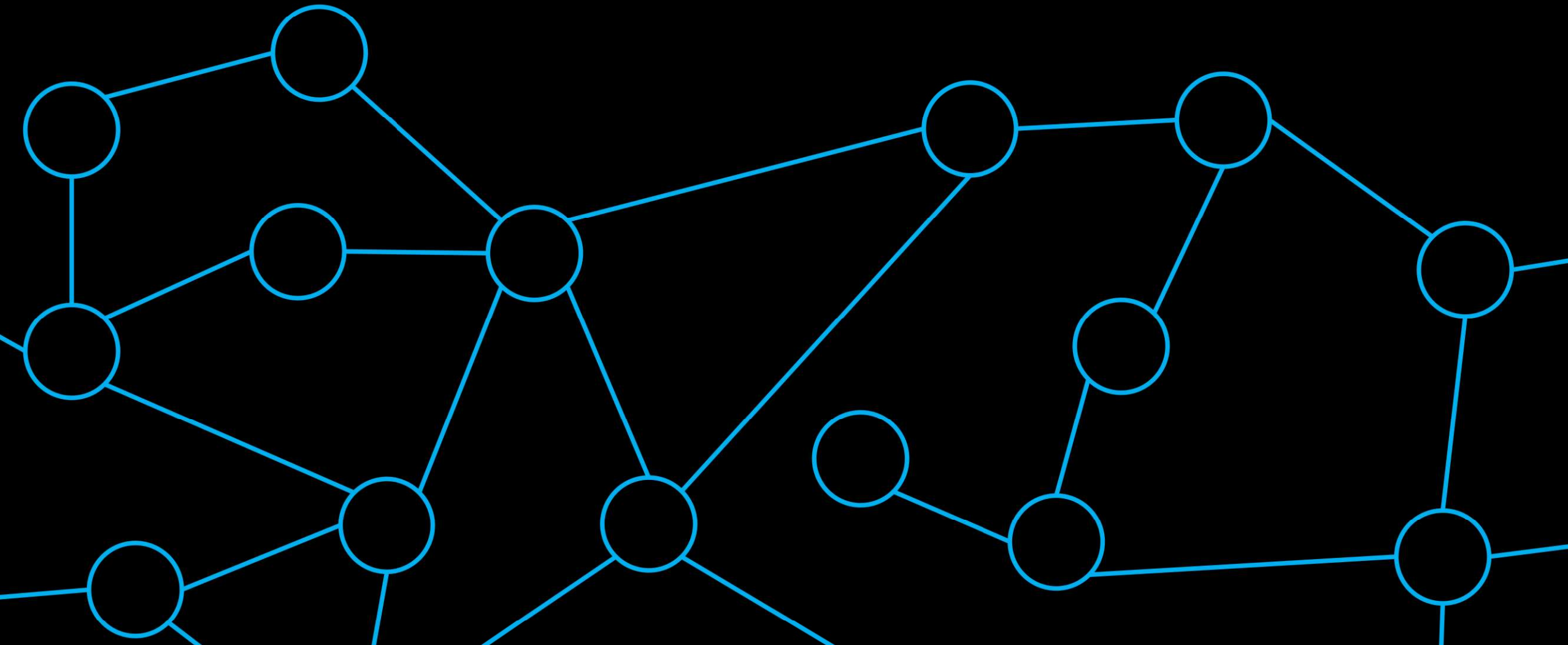
Online Algorithm

First-fit strategy: $n/4$ - competitive in general graphs

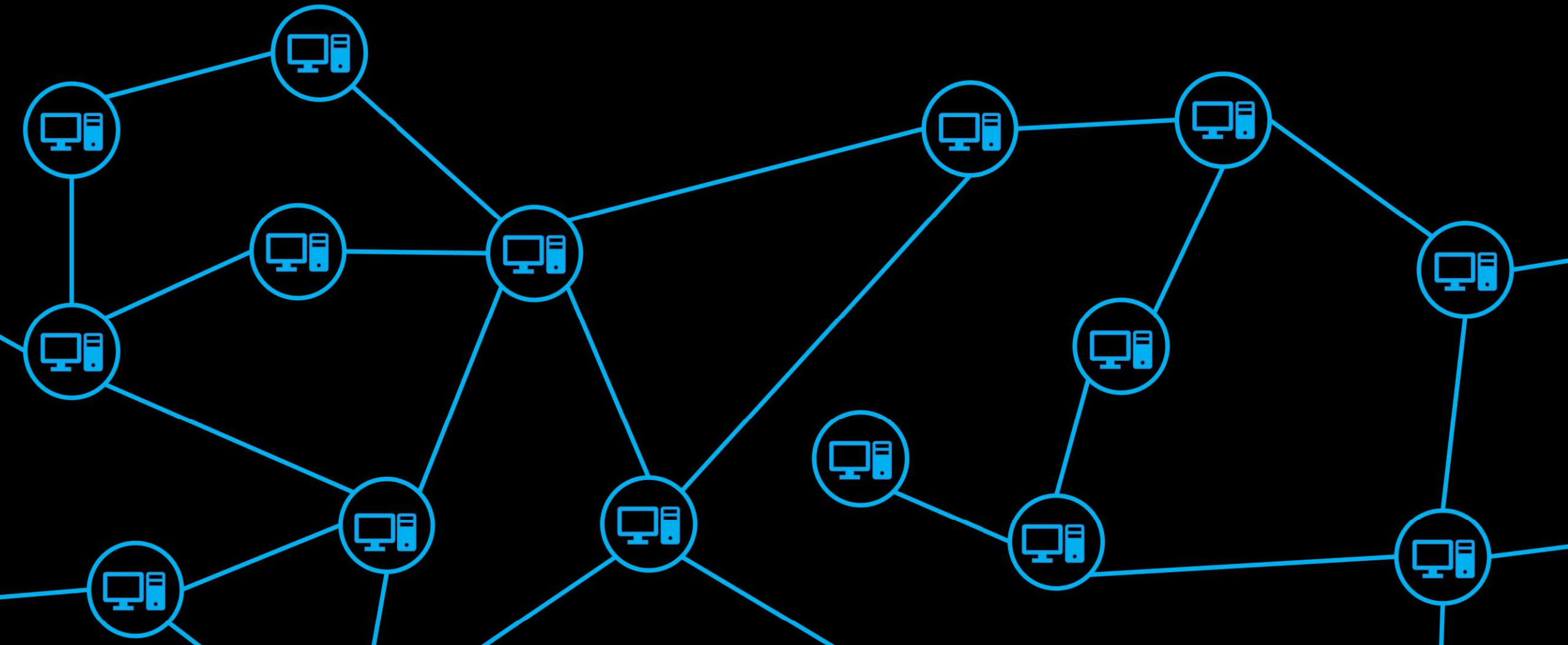
[Lovász et al., 1989]



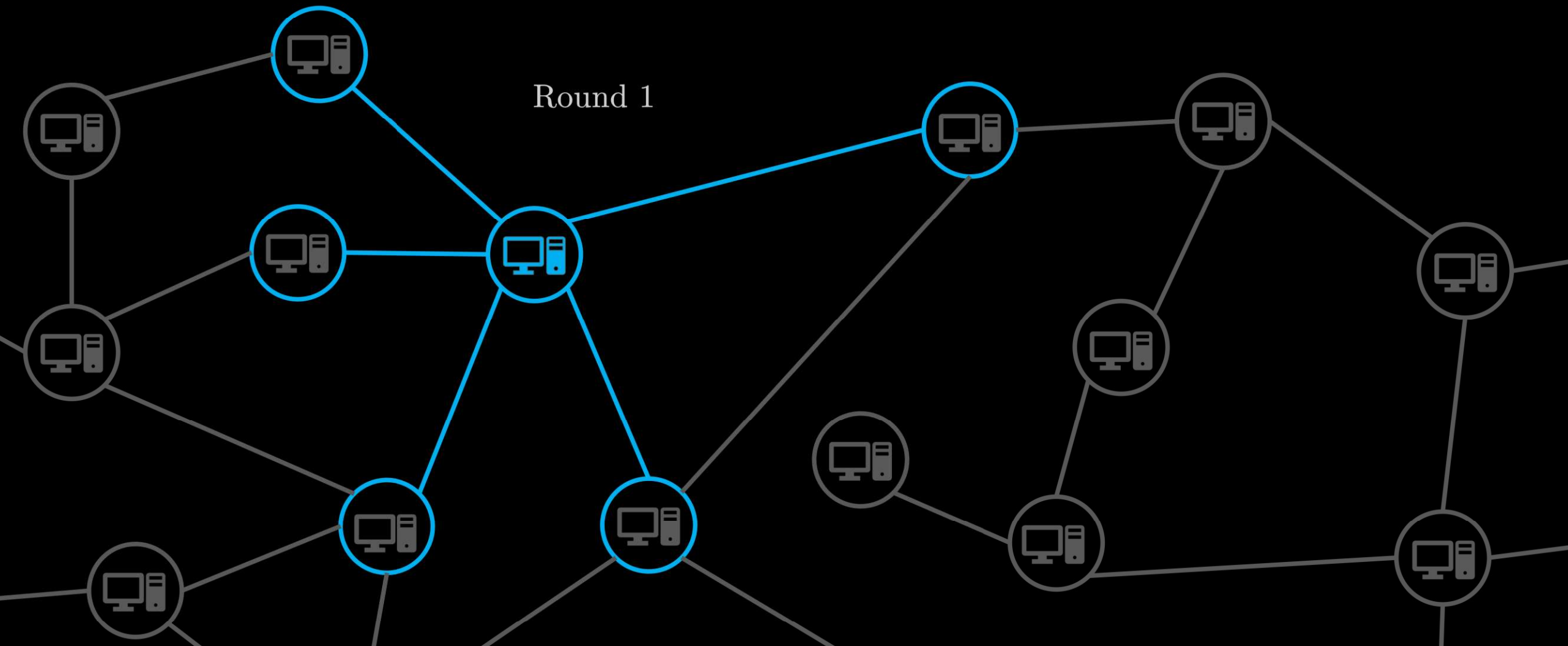
LOCAL Model



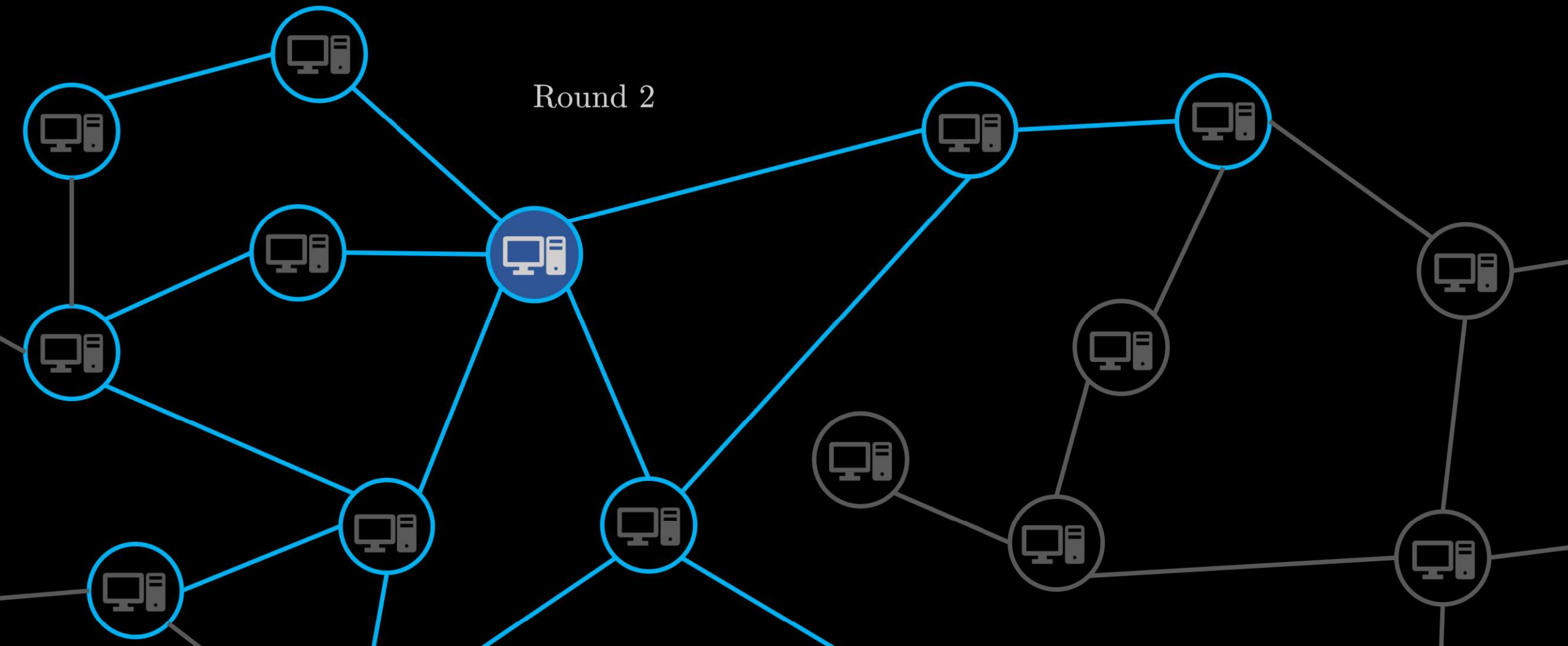
LOCAL Model



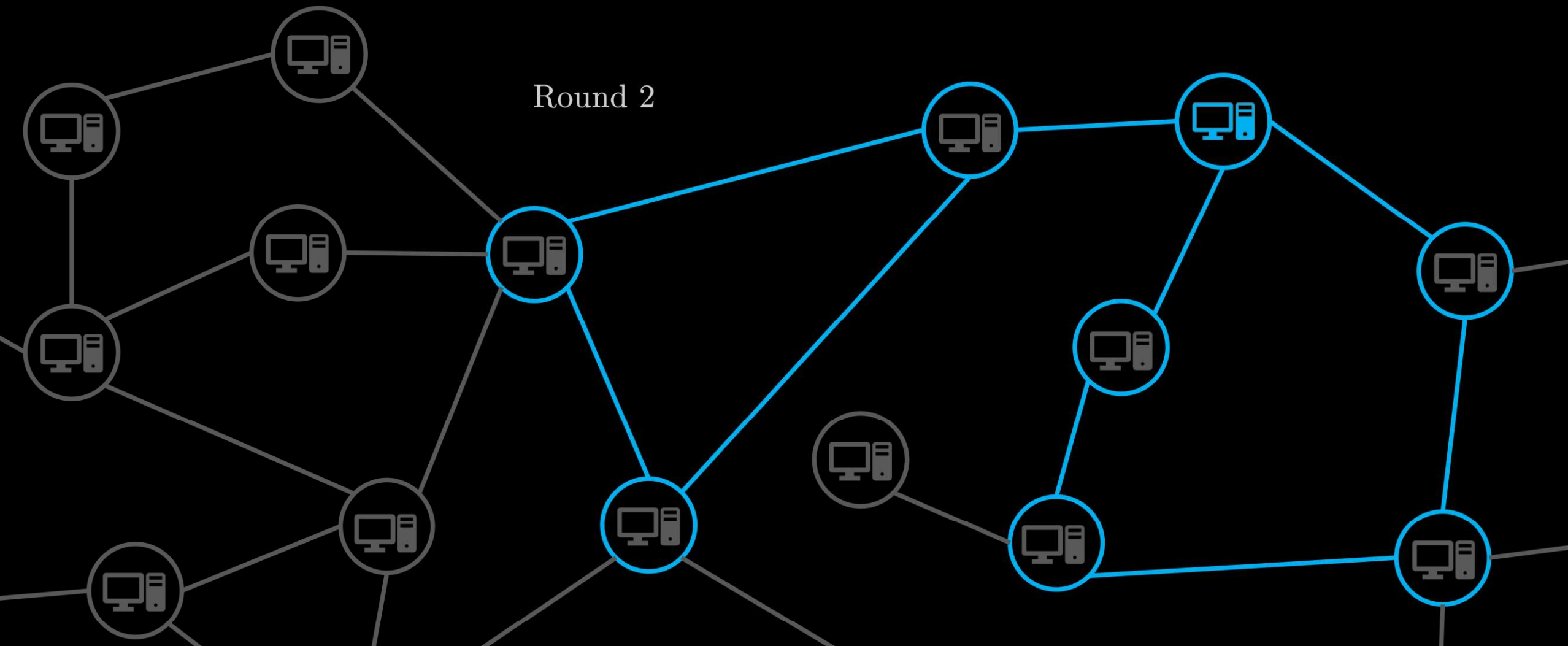
LOCAL Model



LOCAL Model

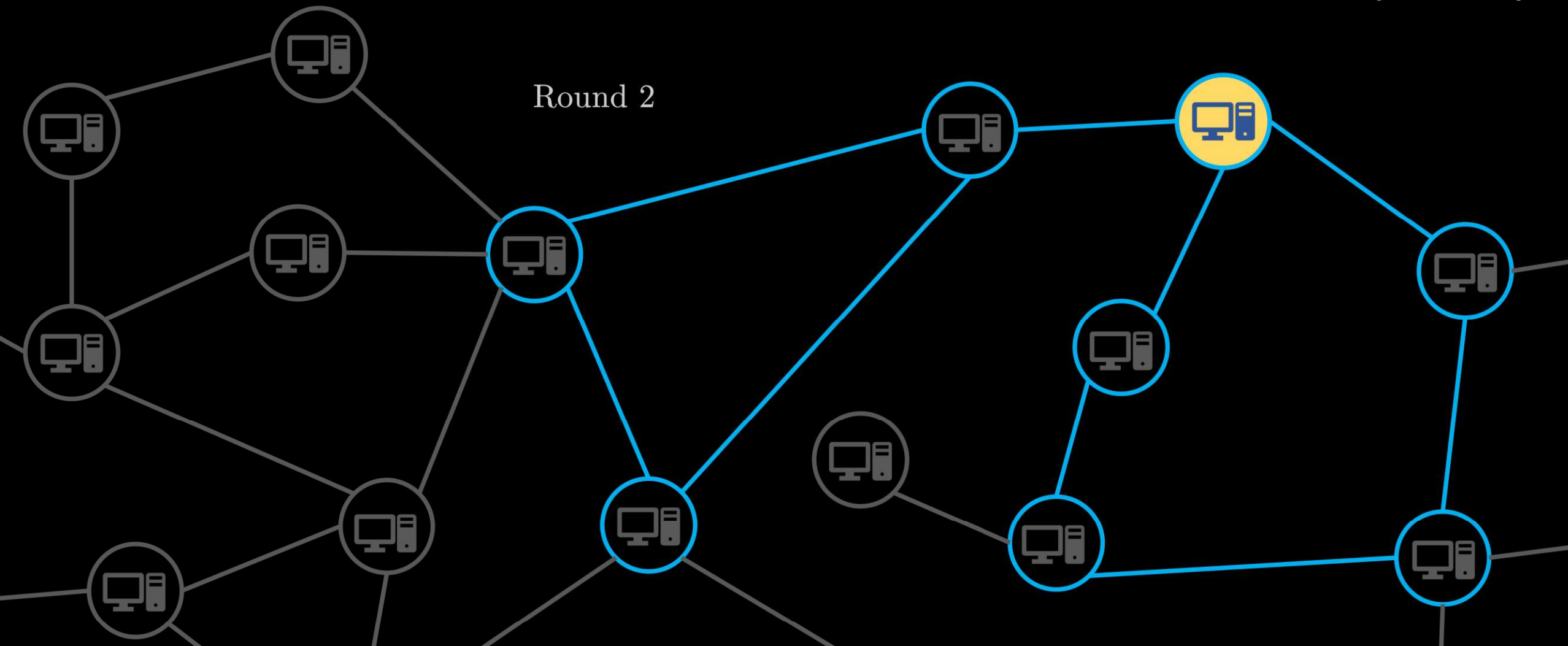


LOCAL Model

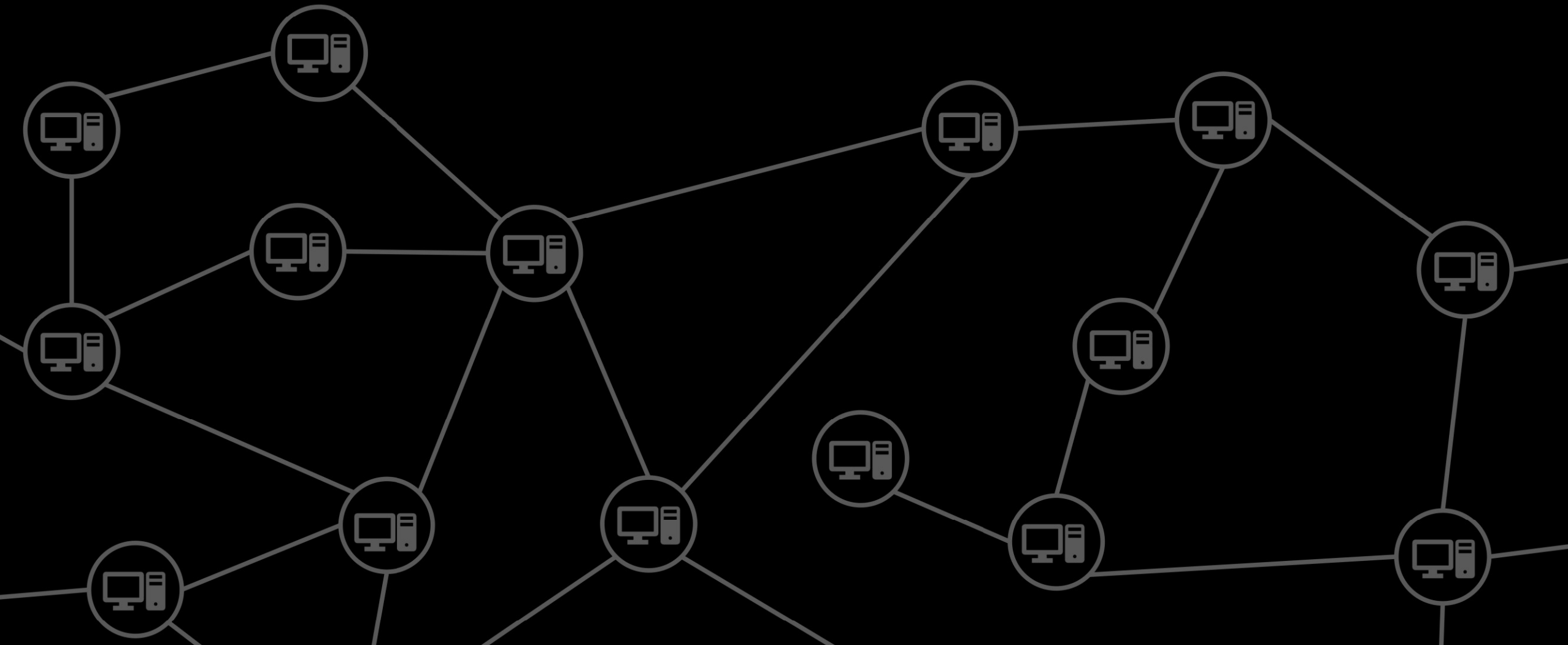


LOCAL Model

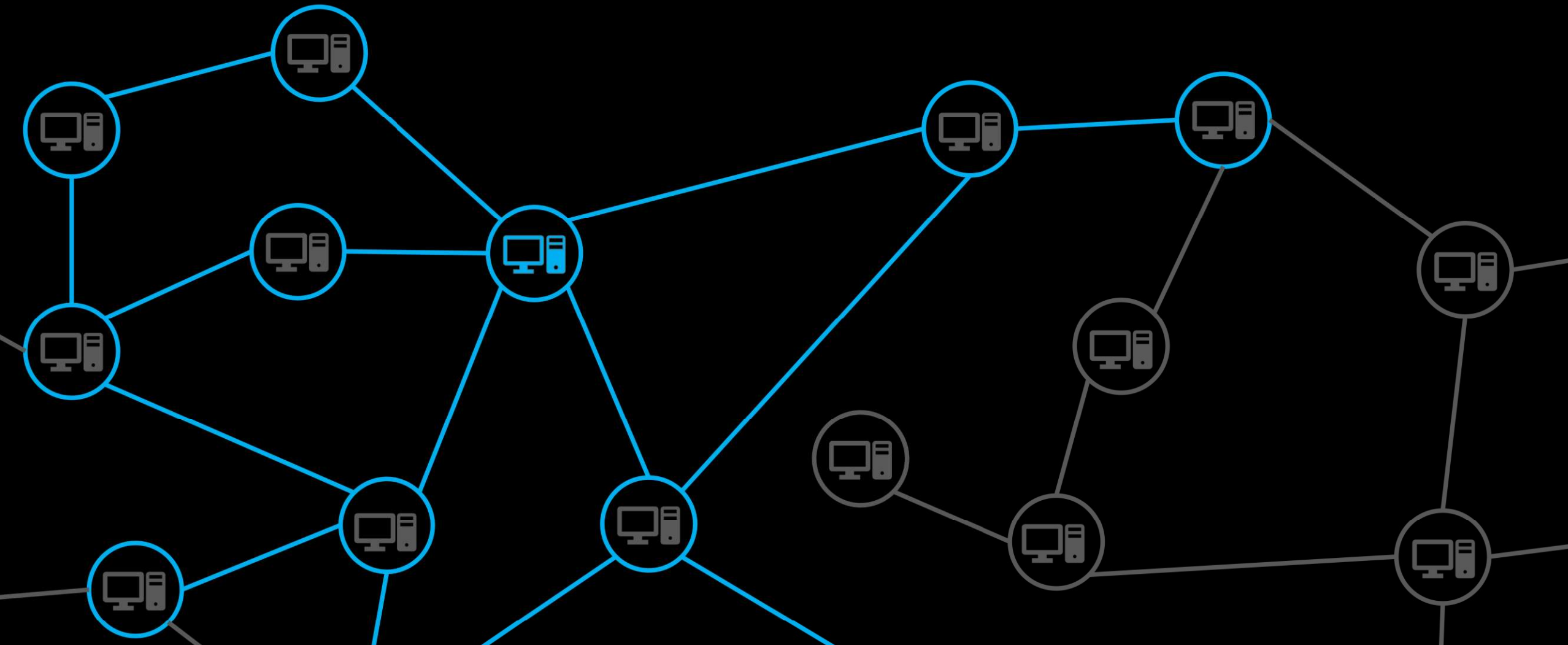
$O(\Delta^2)$-coloring possible in $O(\log^* n)$ rounds
[Linial, 1992]



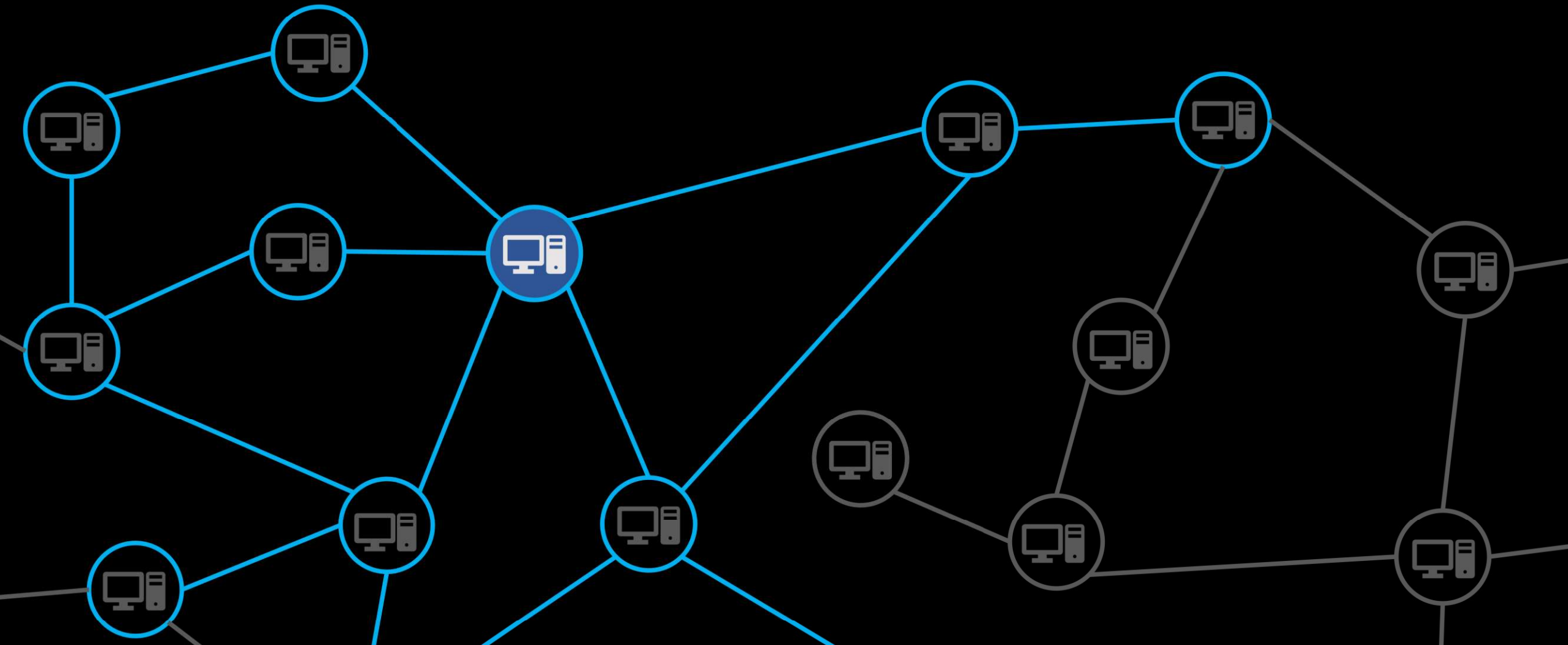
Sequential LOCAL Model (SLOCAL)



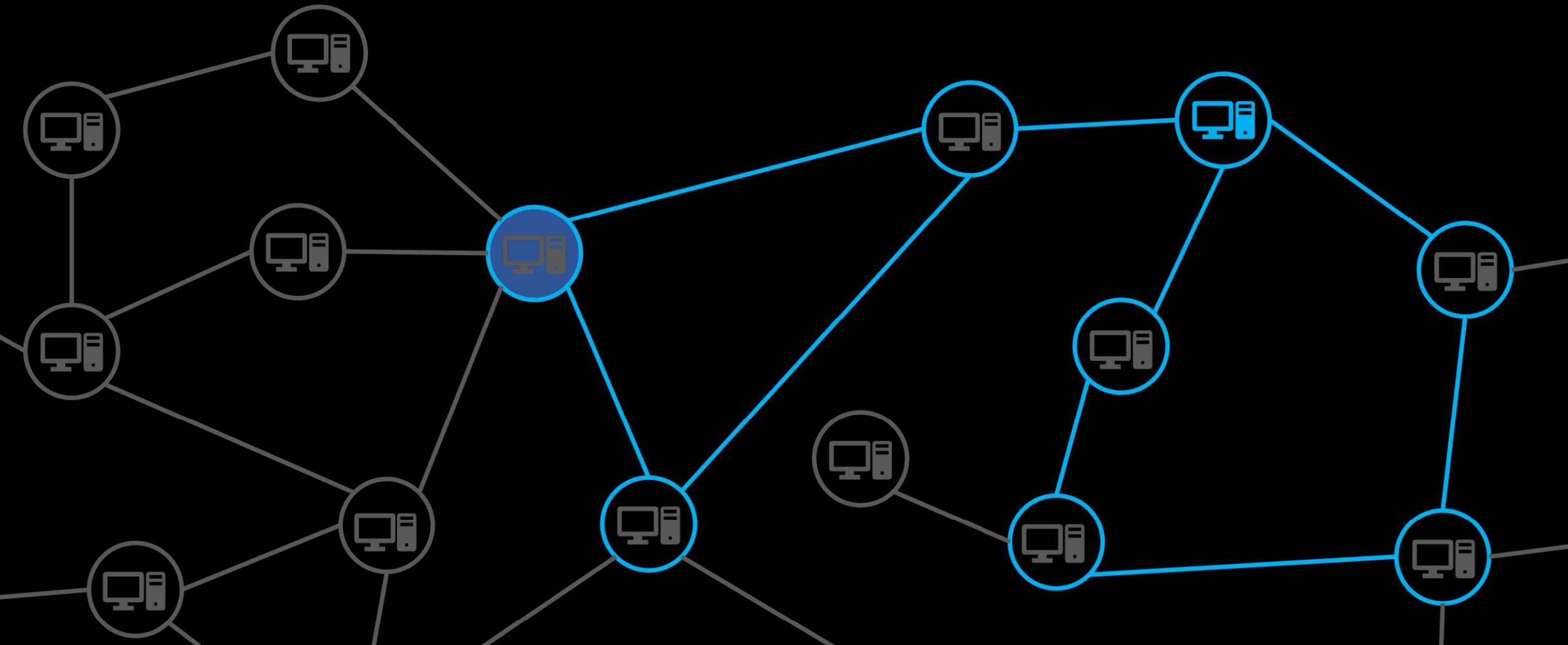
SLOCAL



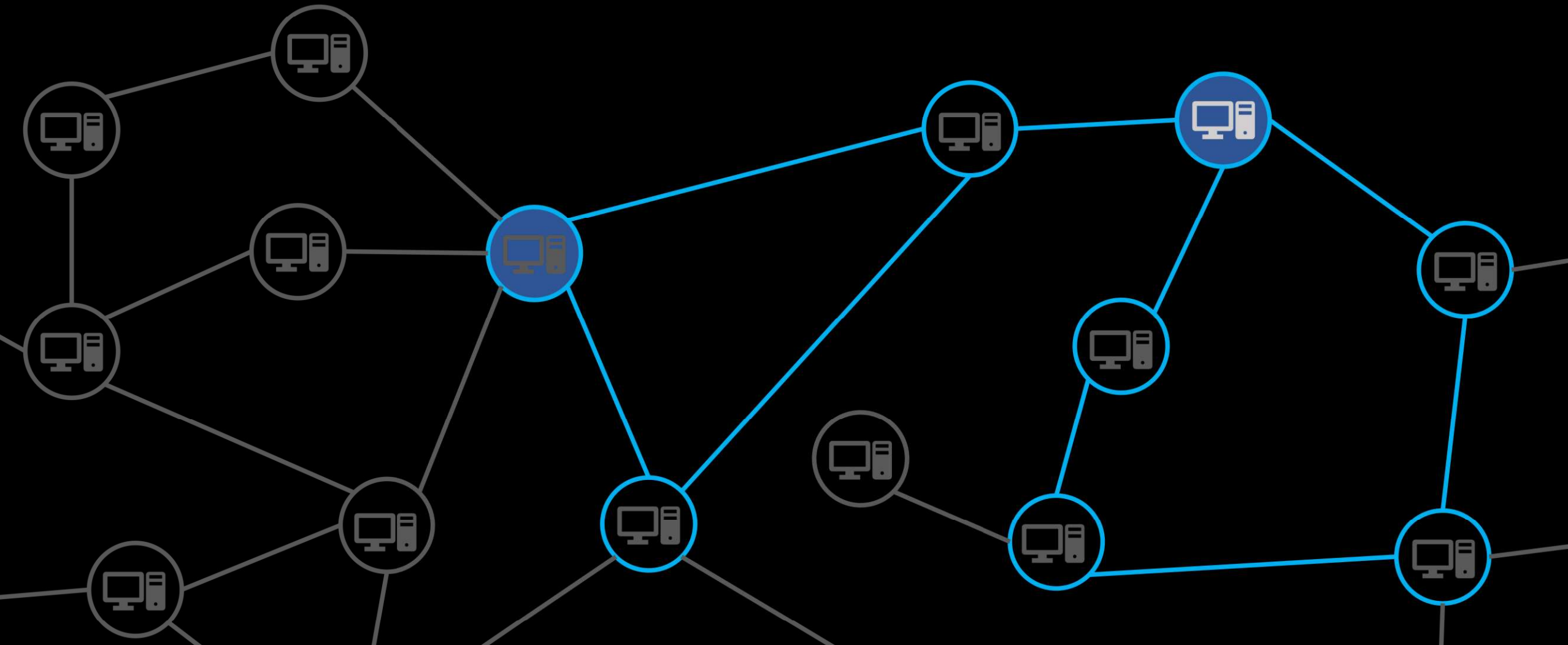
SLOCAL



SLOCAL

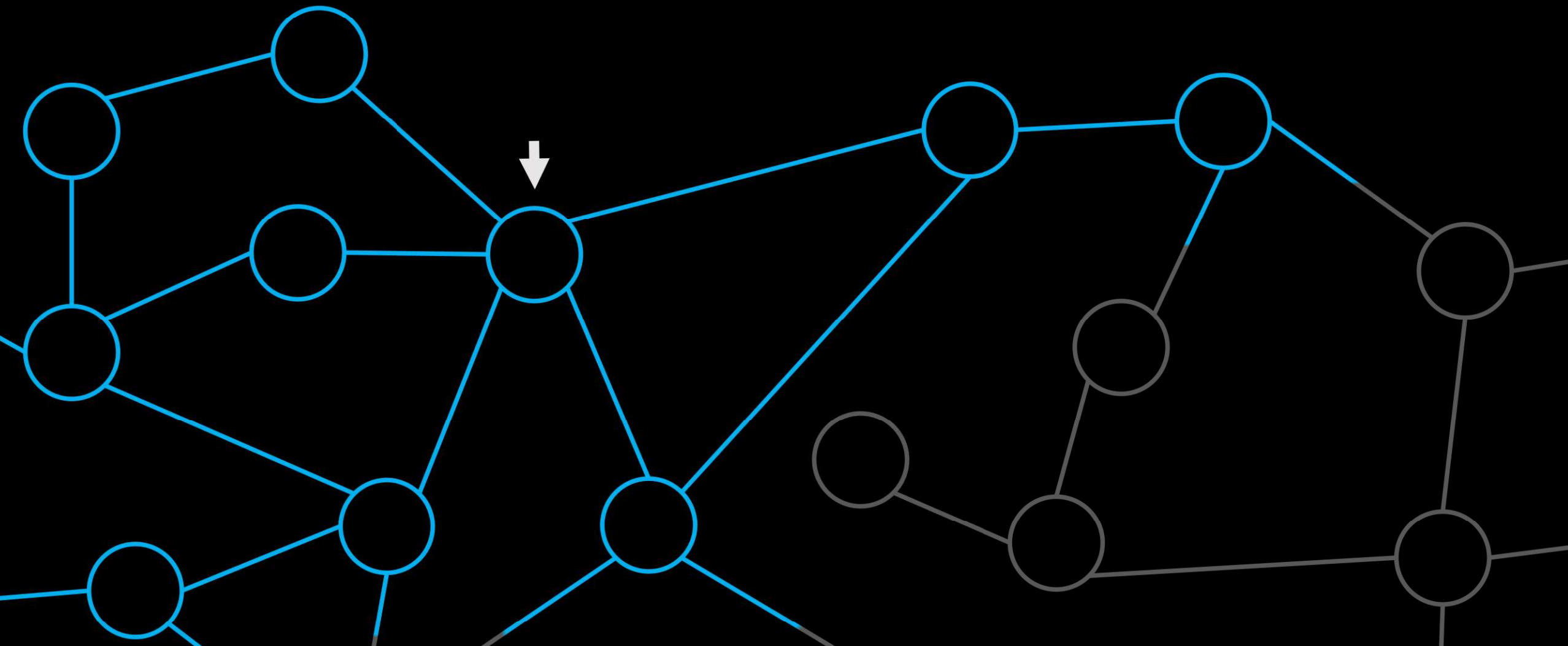


SLOCAL

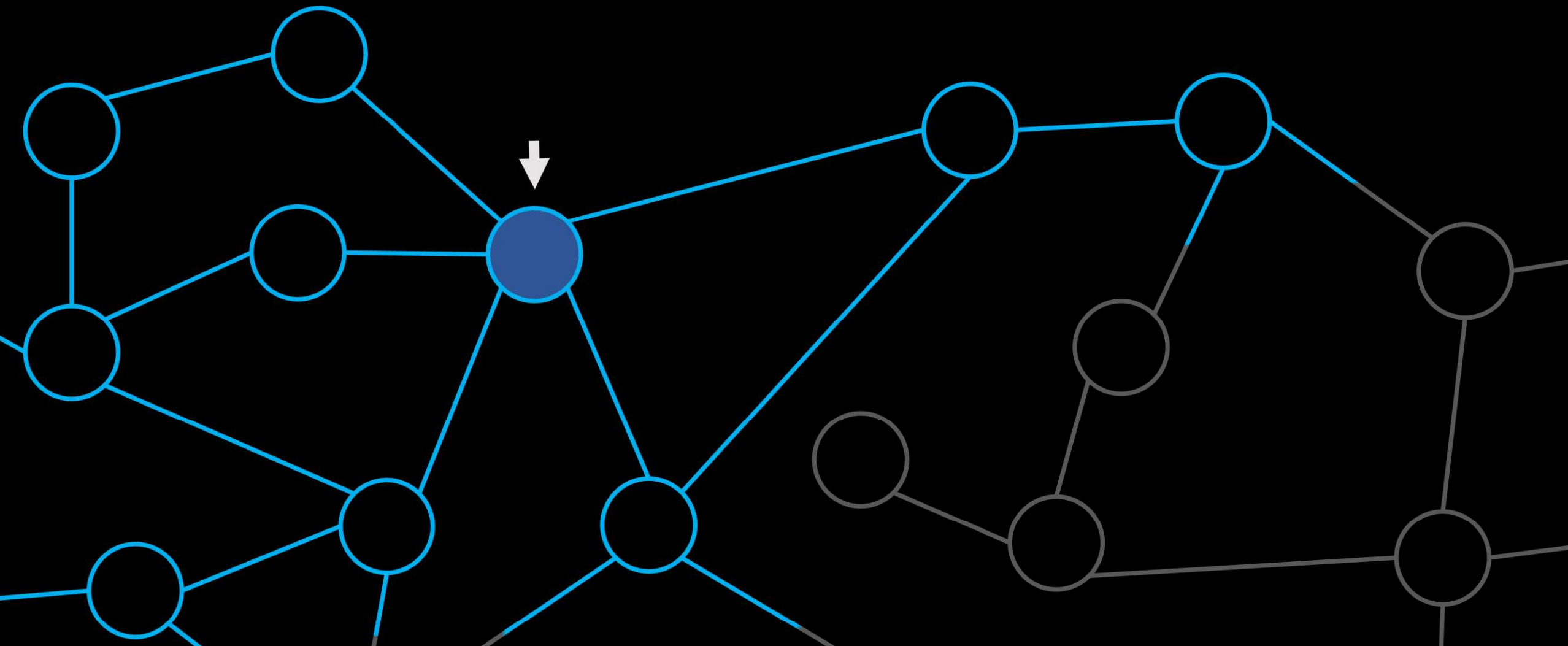


Online Algorithm with Lookaround (online-LOCAL)

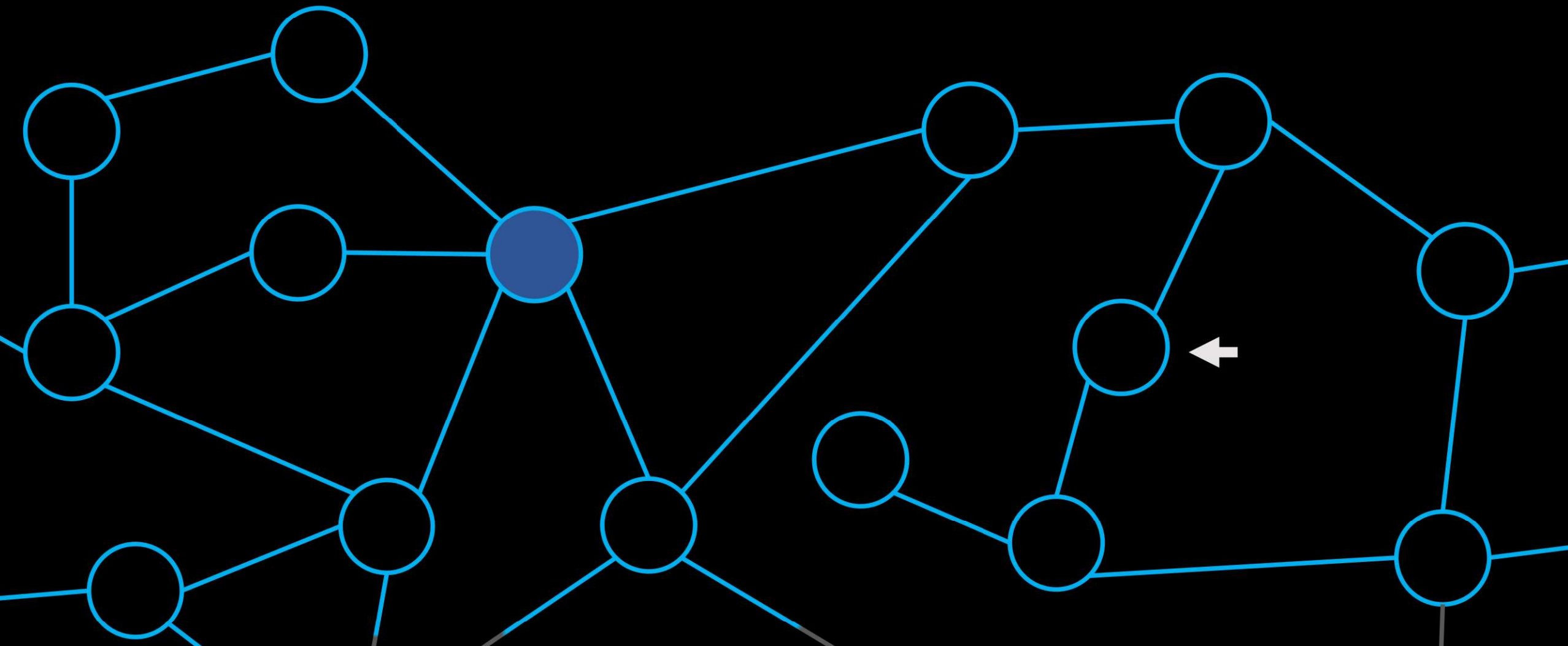
Online-LOCAL



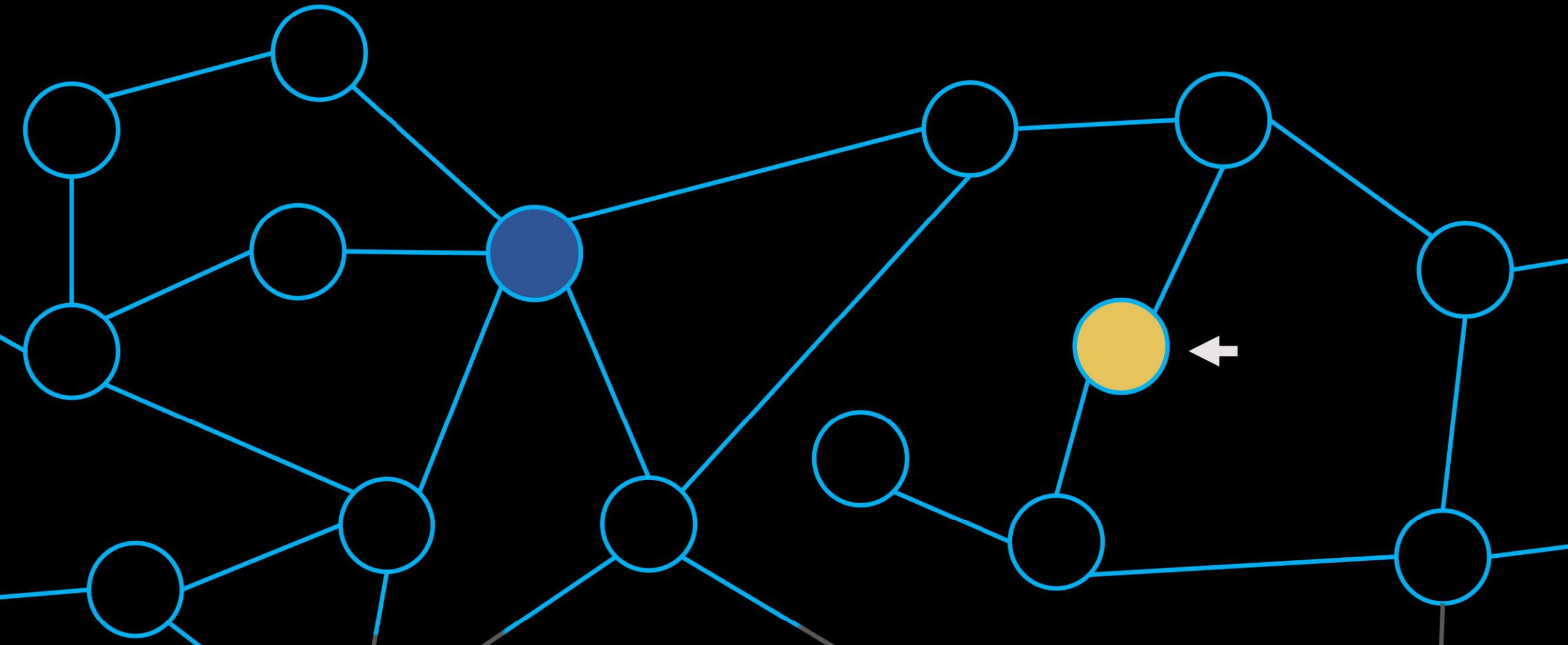
Online-LOCAL



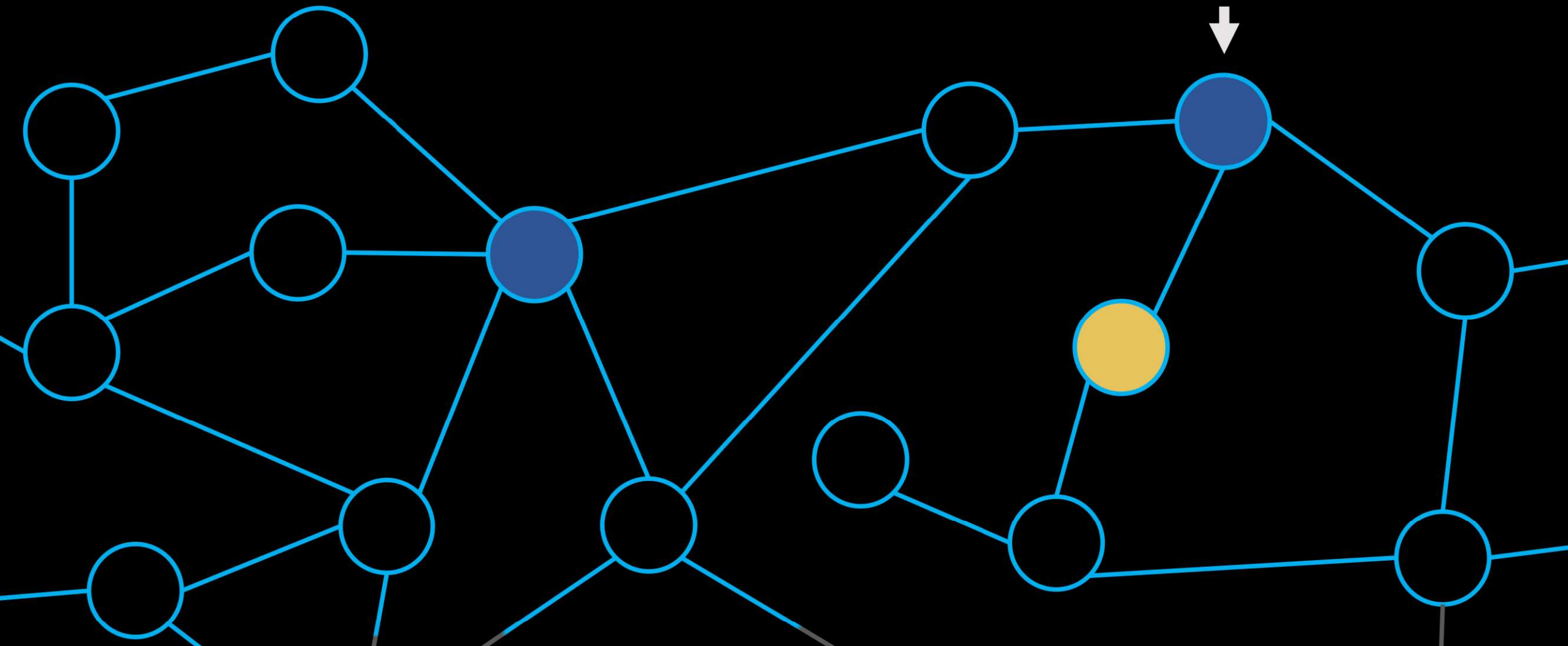
Online-LOCAL



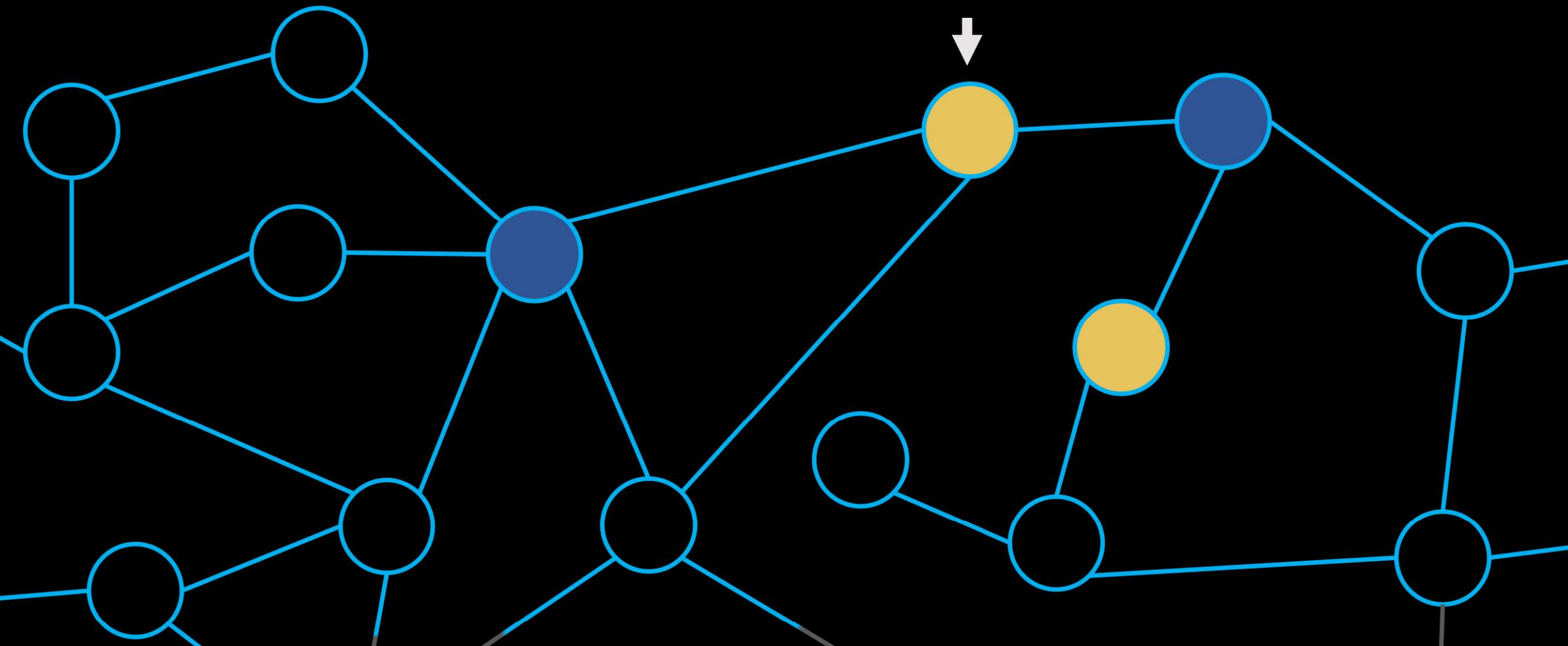
Online-LOCAL



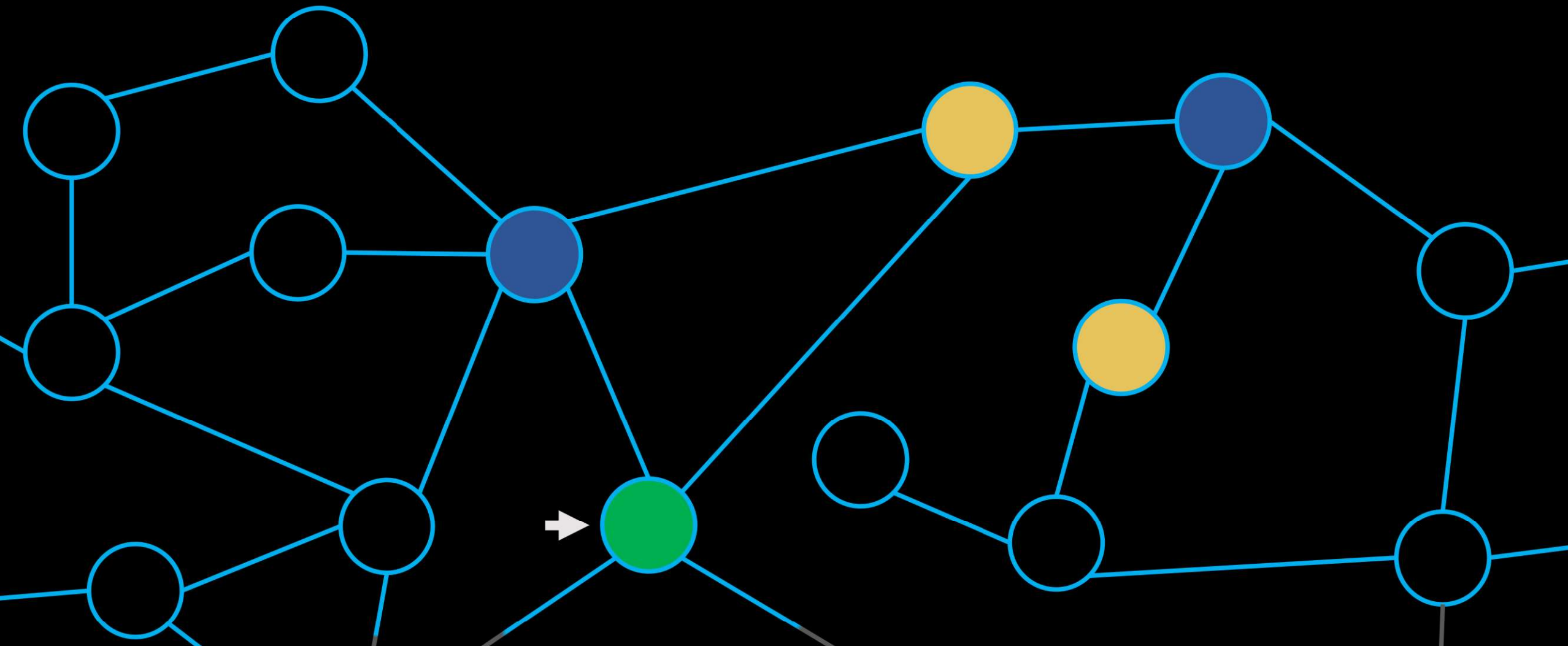
Online-LOCAL



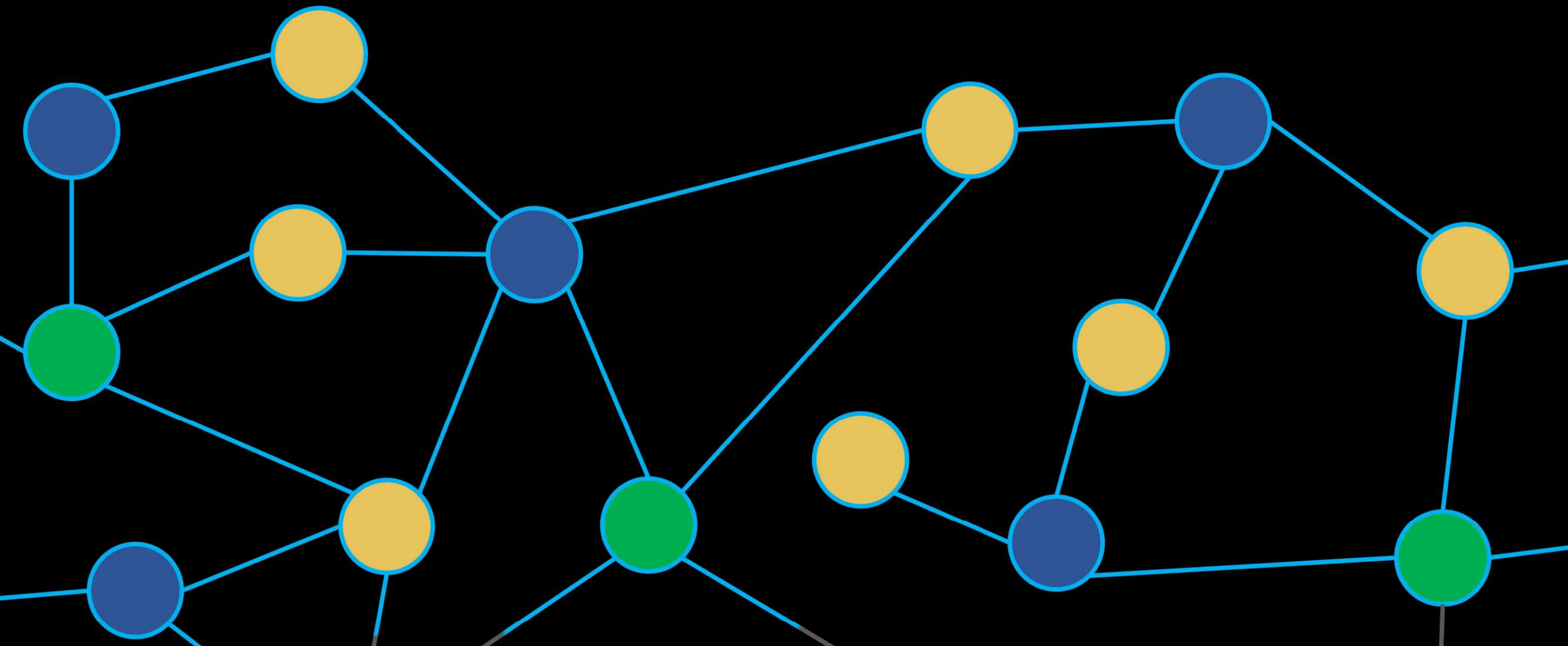
Online-LOCAL



Online-LOCAL



Online-LOCAL



Interpretation of online-LOCAL

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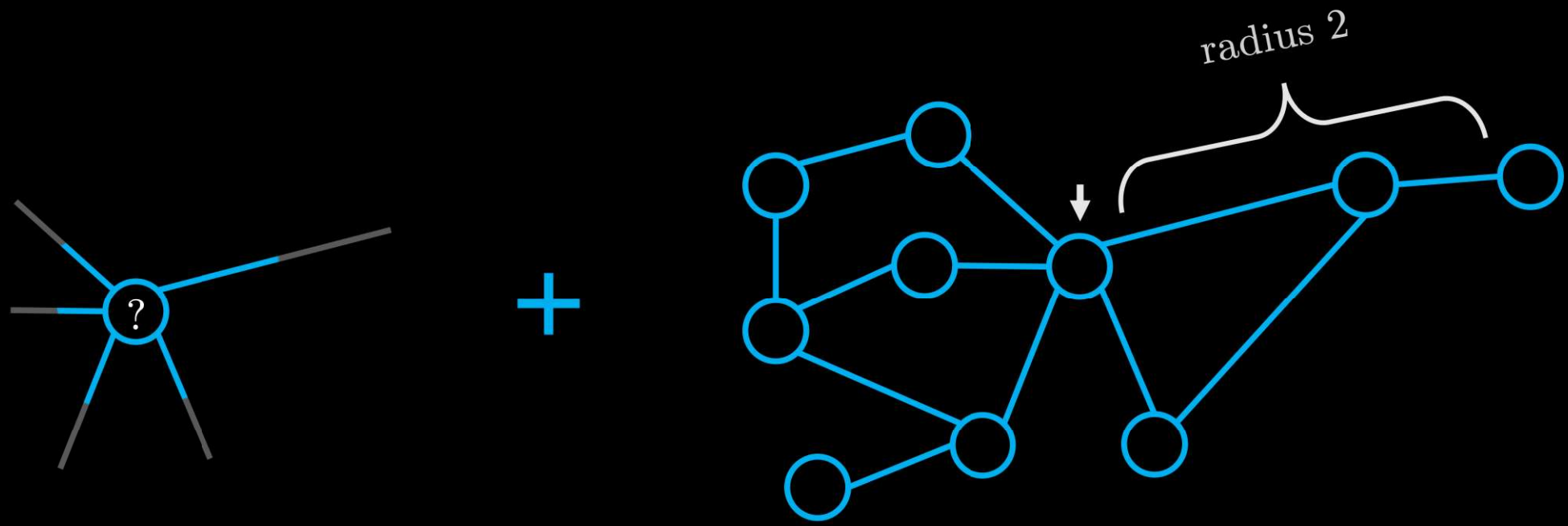
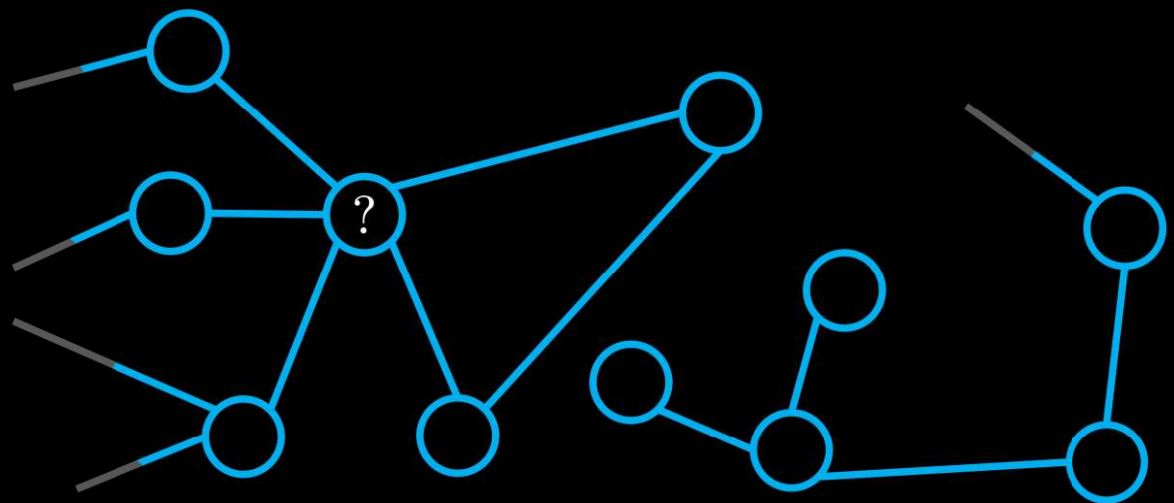
- Lookaround as advice



Interpretation of online-LOCAL

- Lookaround as advice
- Delaying the decision



Interpretation of online-LOCAL

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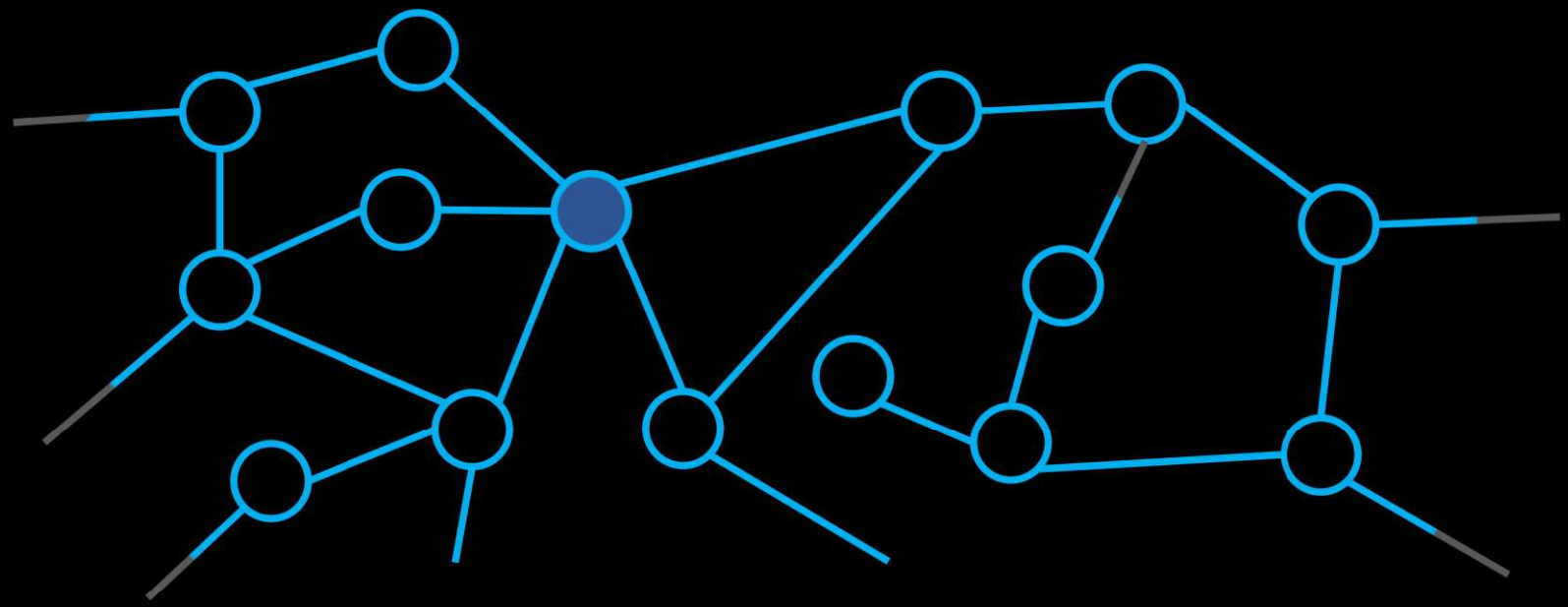
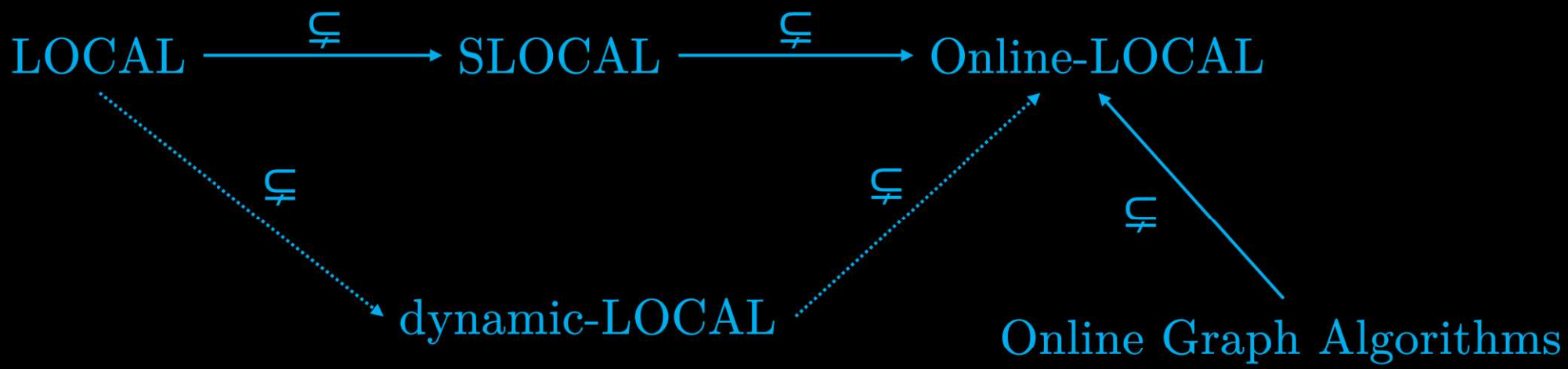
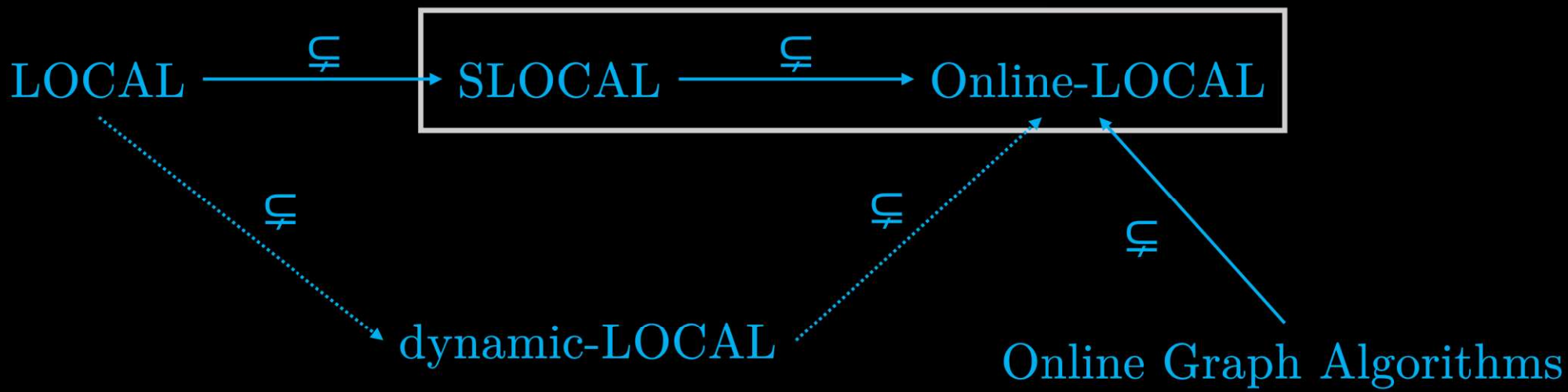


Interpretation of online-LOCAL

- Lookaround as advice
- Delaying the decision
- Connection between distributed and online algorithms




LOCAL
⊊
SLOCAL
⊊
Online-LOCAL
⊊
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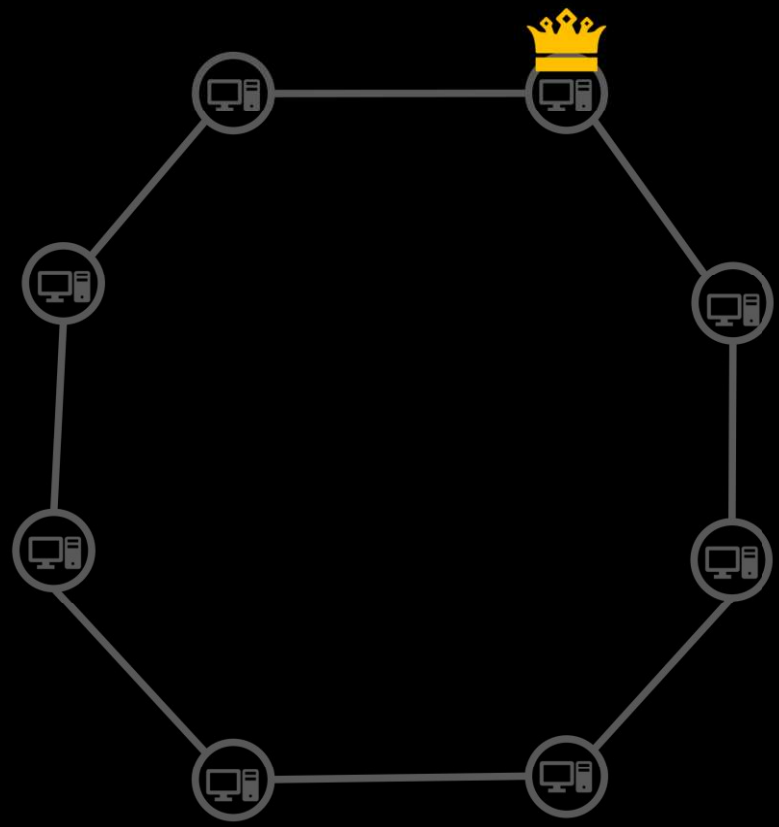
⊊
dynamic-LOCAL
Online Graph Algorithms

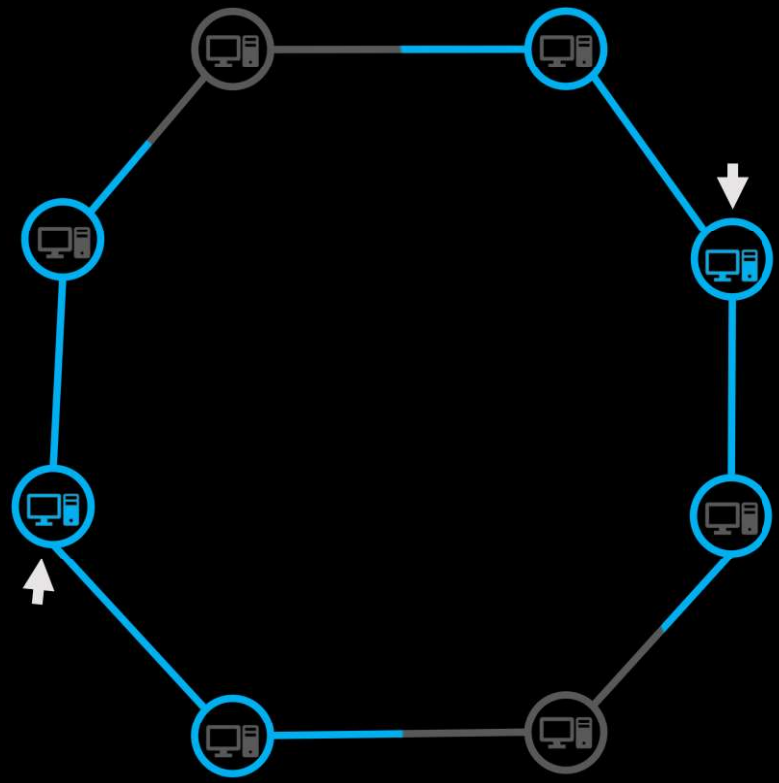
SLOCAL $\subsetneq$ Online-LOCAL

Component-wise Leader Election



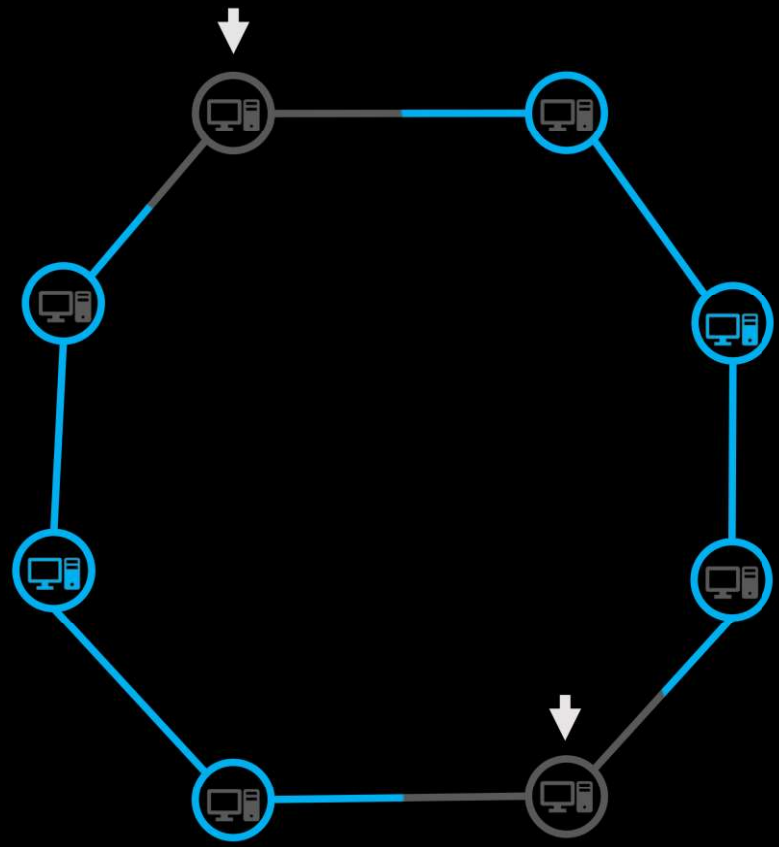
SLOCAL $\subsetneq$ Online-LOCAL

👑 Component-wise Leader Election



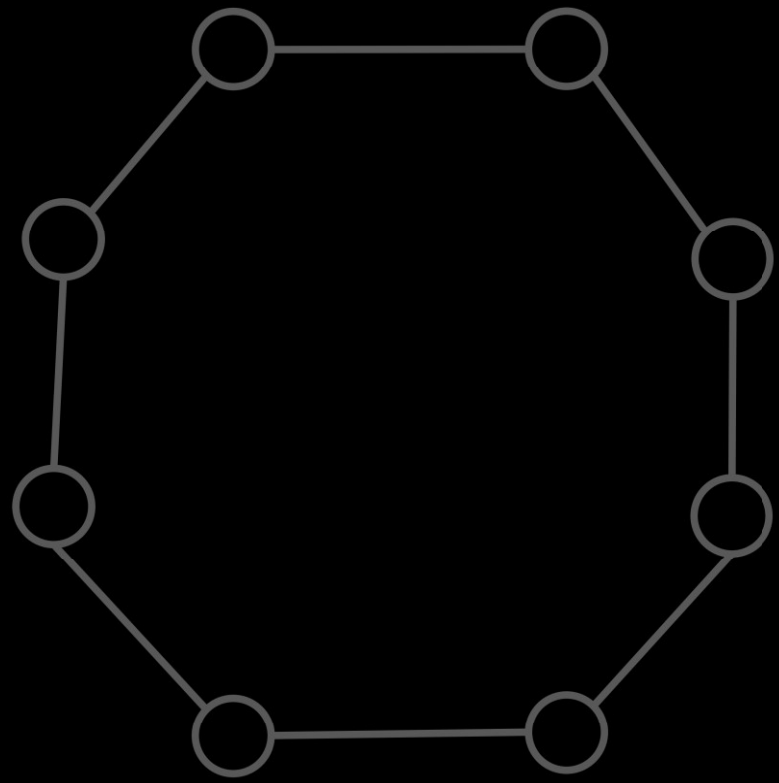
SLOCAL $\subsetneq$ Online-LOCAL

👑 Component-wise Leader Election



SLOCAL $\subsetneq$ Online-LOCAL

👑 Component-wise Leader Election



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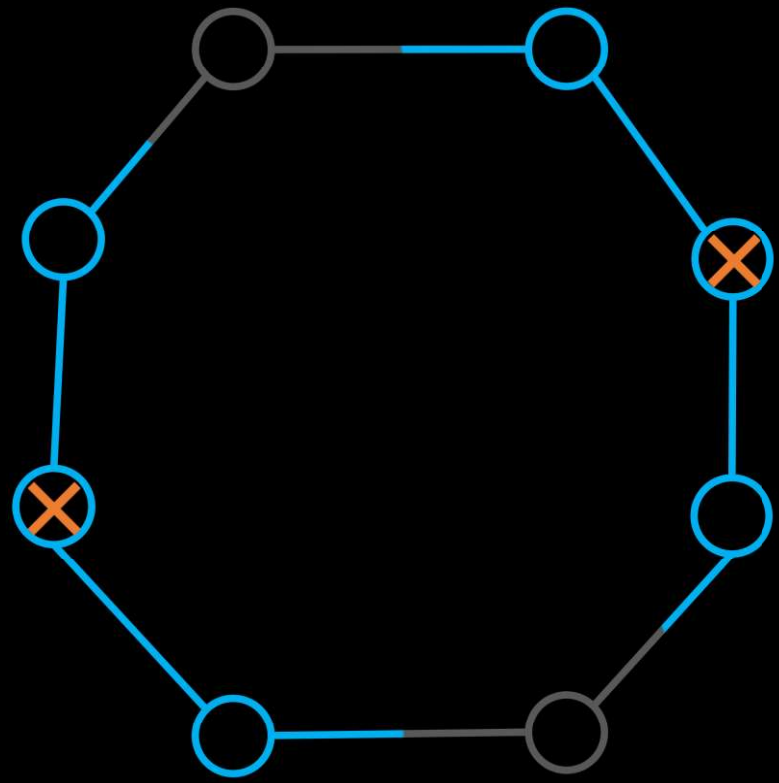

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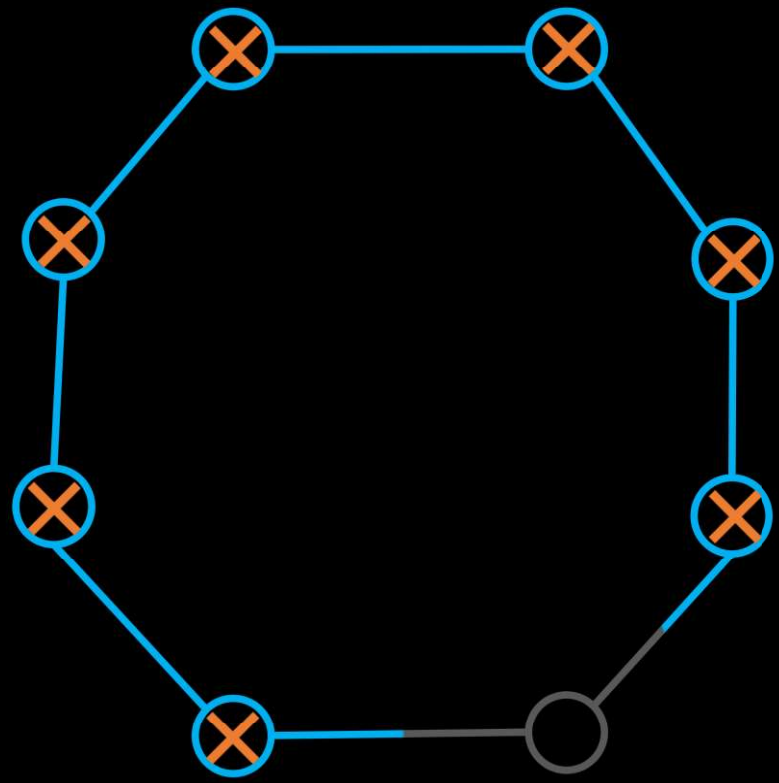
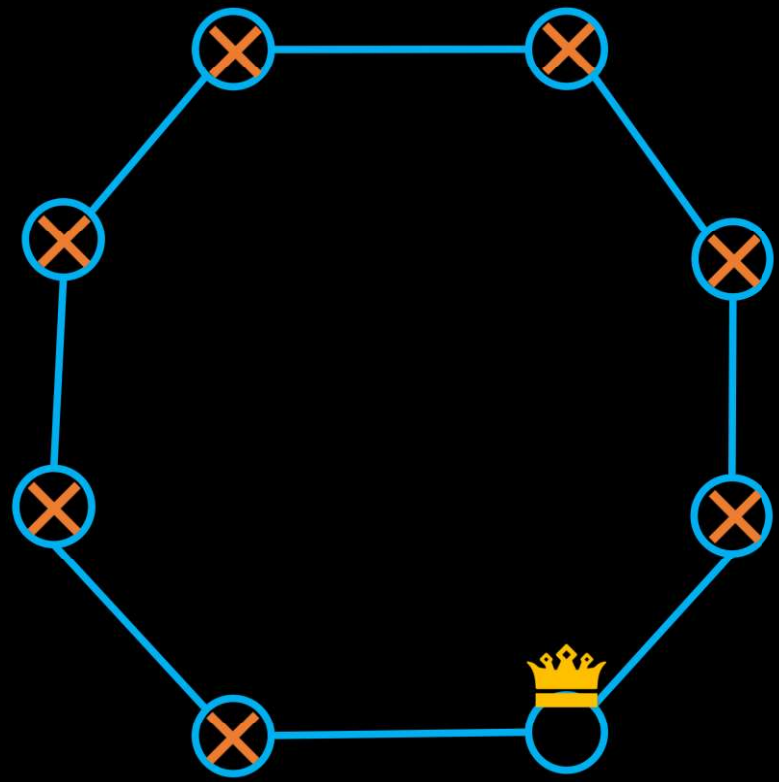
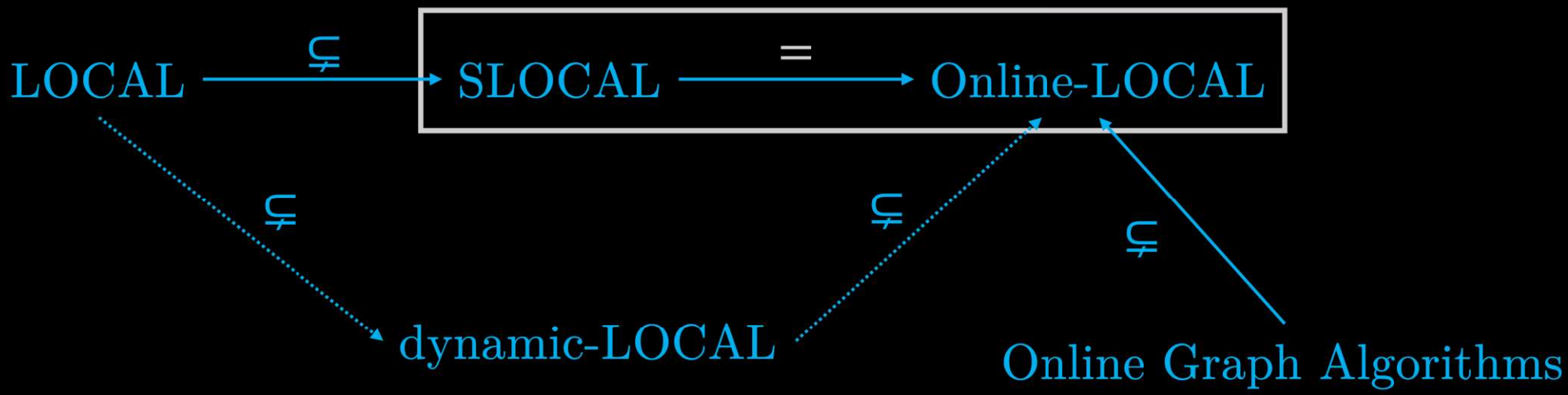


SLOCAL $\subsetneq$ Online-LOCAL

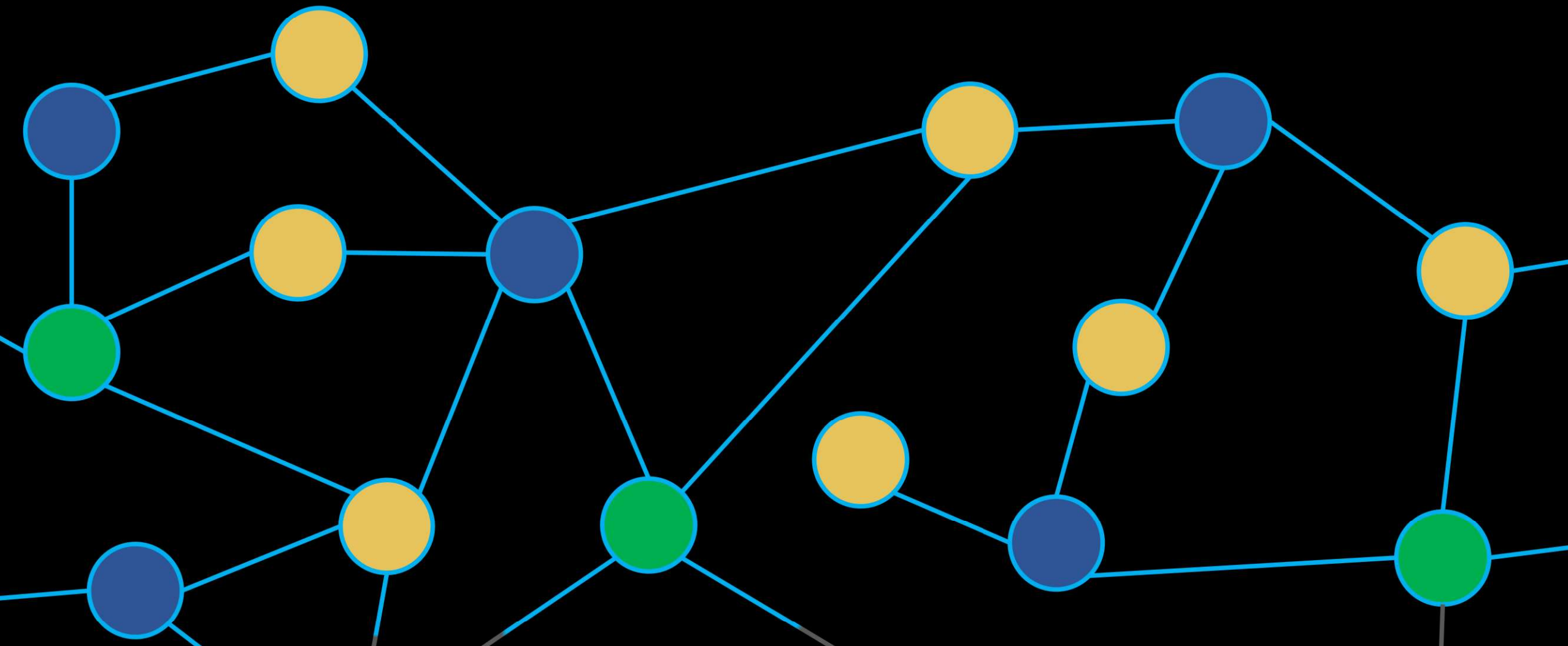
Component-wise Leader Election



For LCLs in paths, cycles, and rooted trees:



LCLs – locally checkable labeling problems



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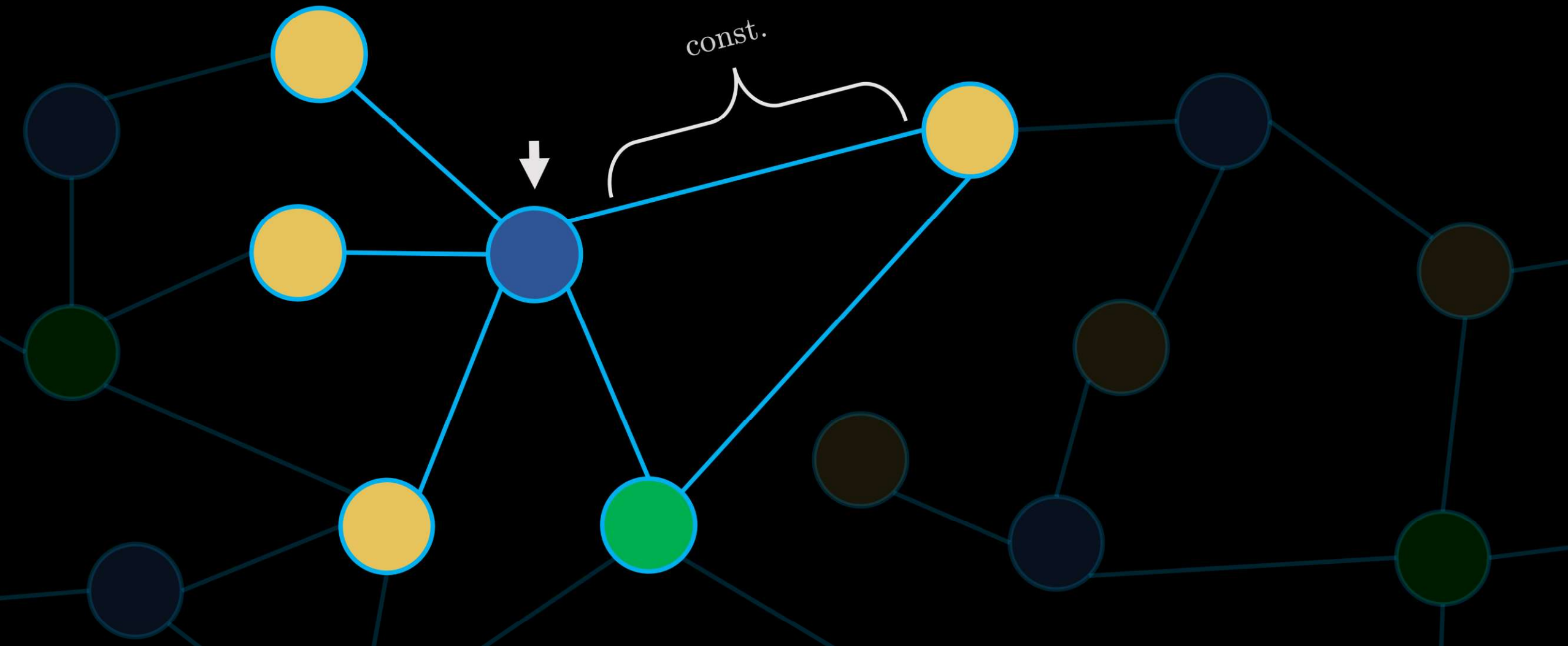
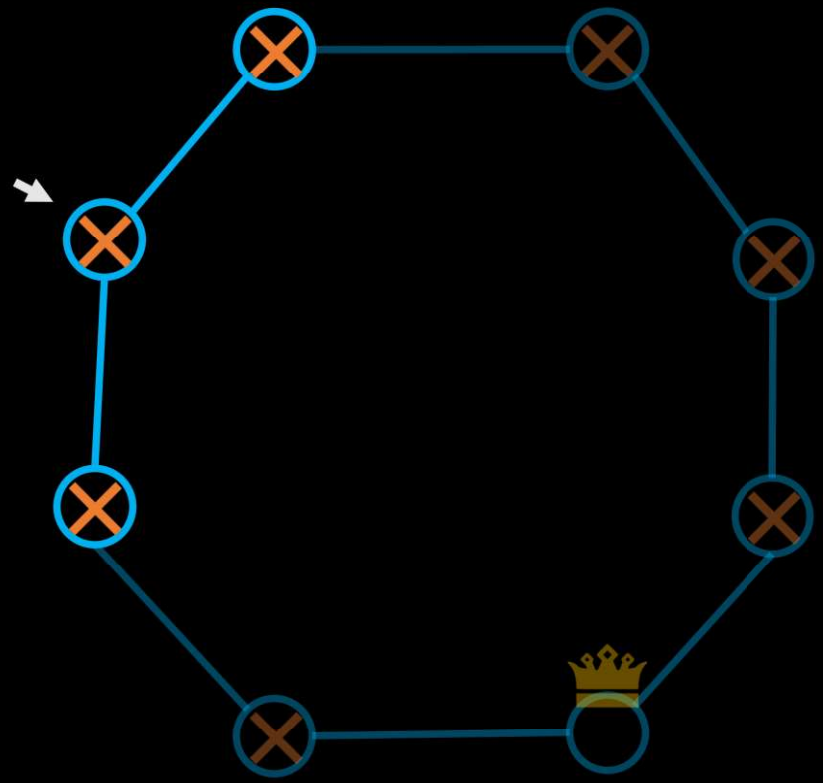


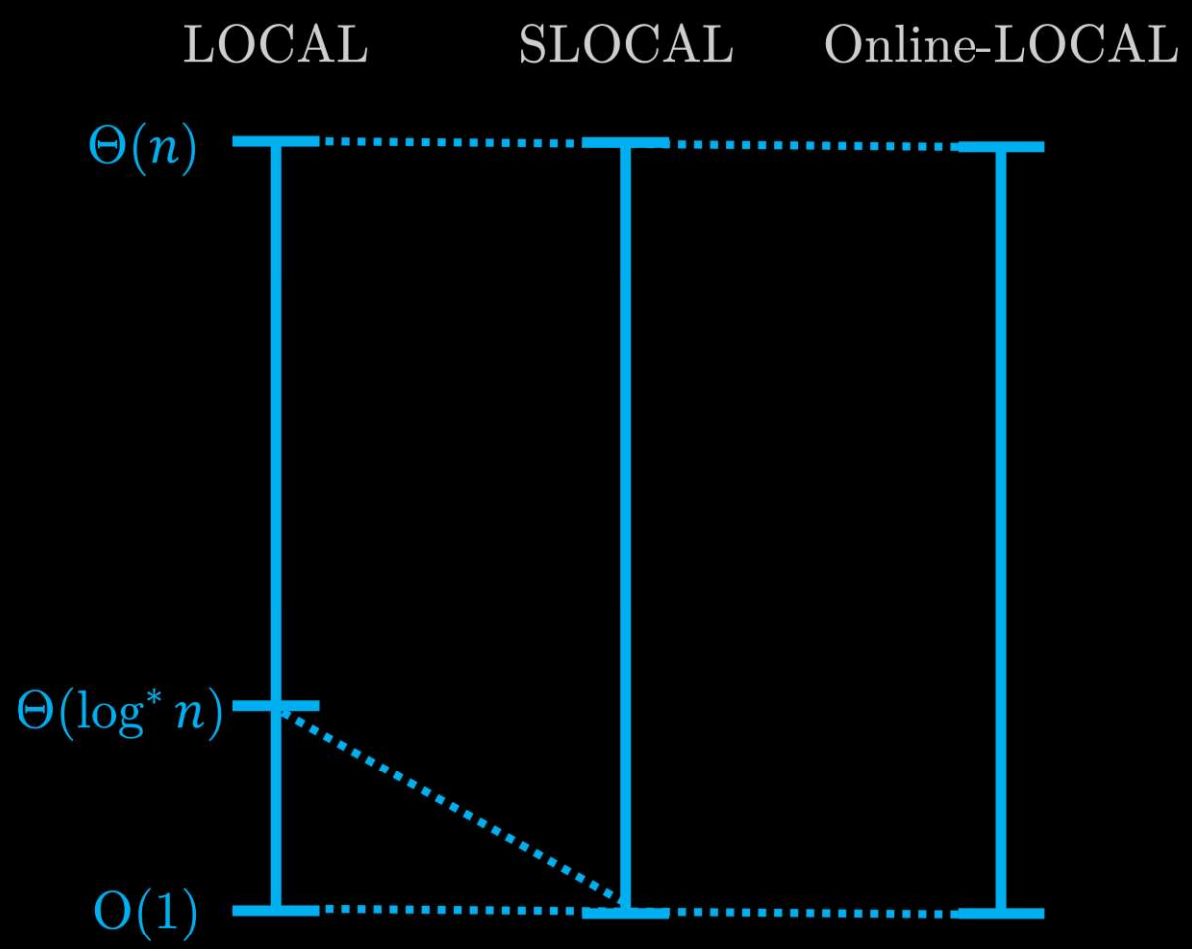
LCLs – locally checkable labeling problems

- Graph Coloring is an LCL
- Component-wise leader election is not an LCL:

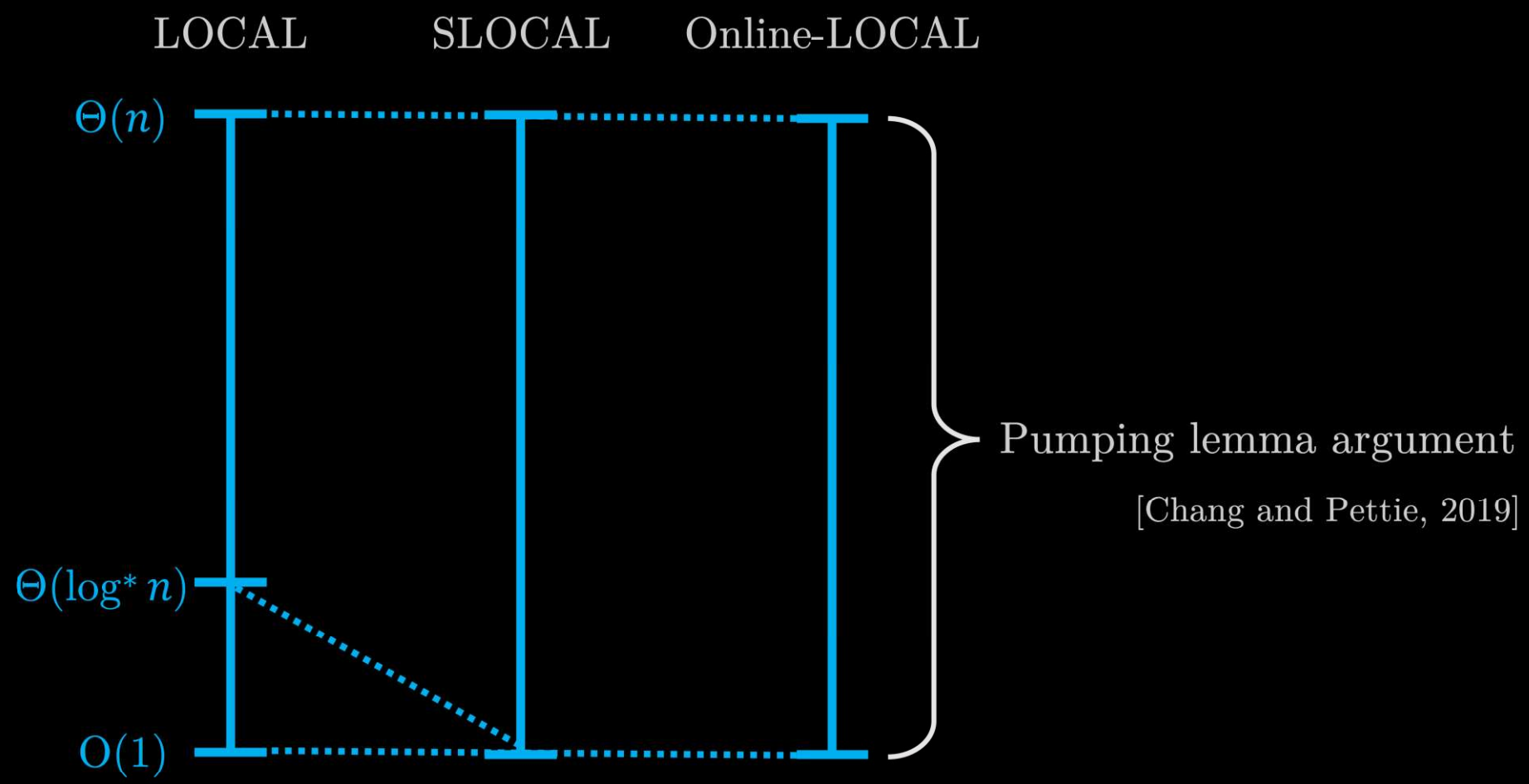


Equivalence proofs for LCLs

LCLs in paths and cycles

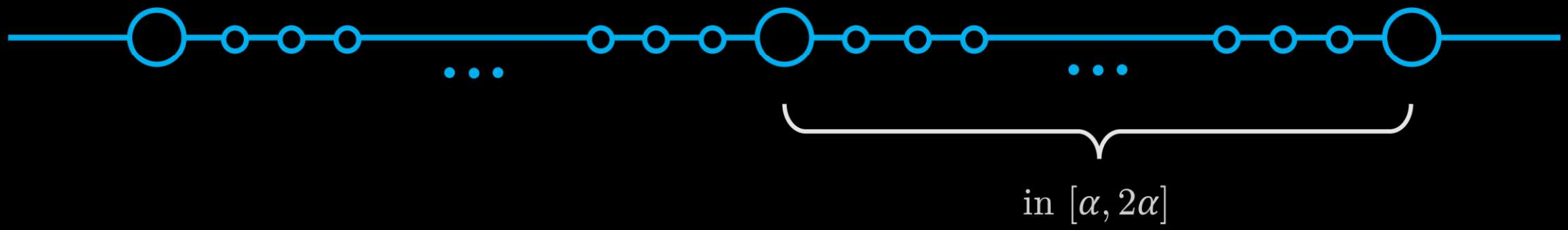


LCLs in paths and cycles



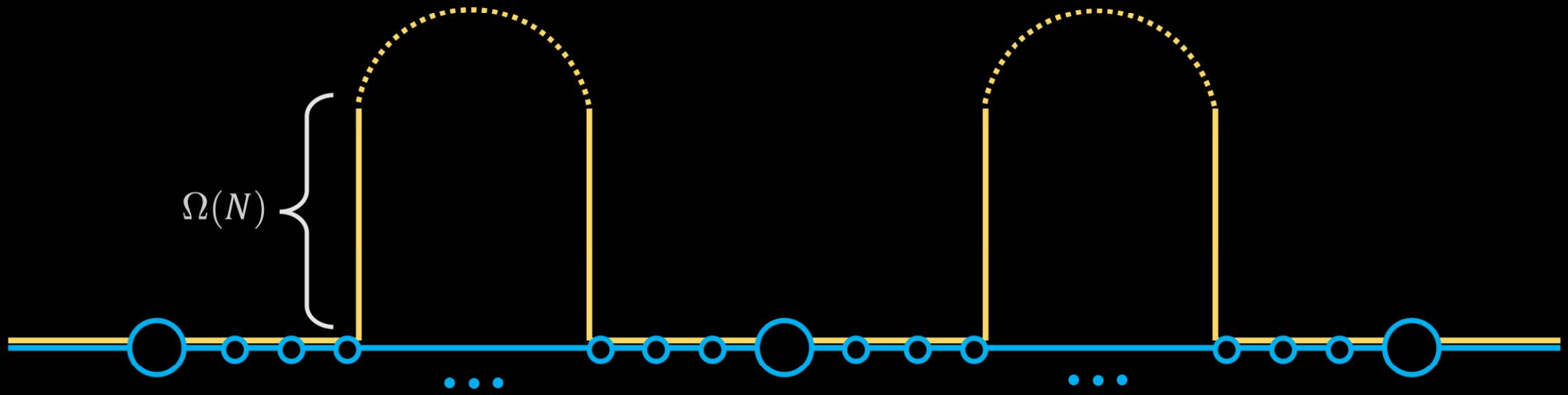
Gap result on paths in online-LOCAL

- Given: online-LOCAL algorithm with $o(n)$-locality



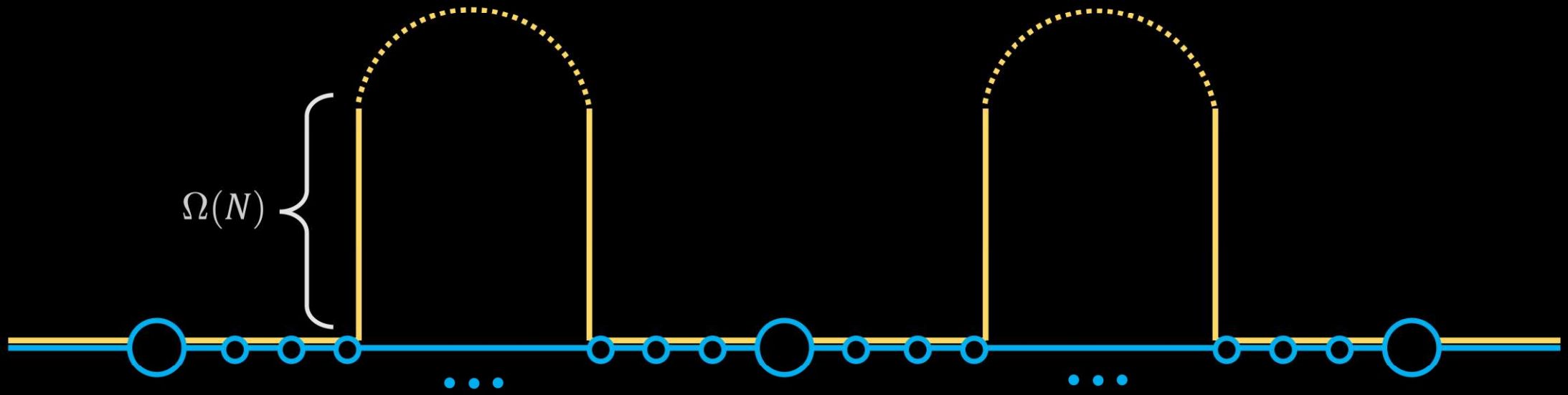
Gap result on paths in online-LOCAL

- Construct new path with N ≫ n nodes



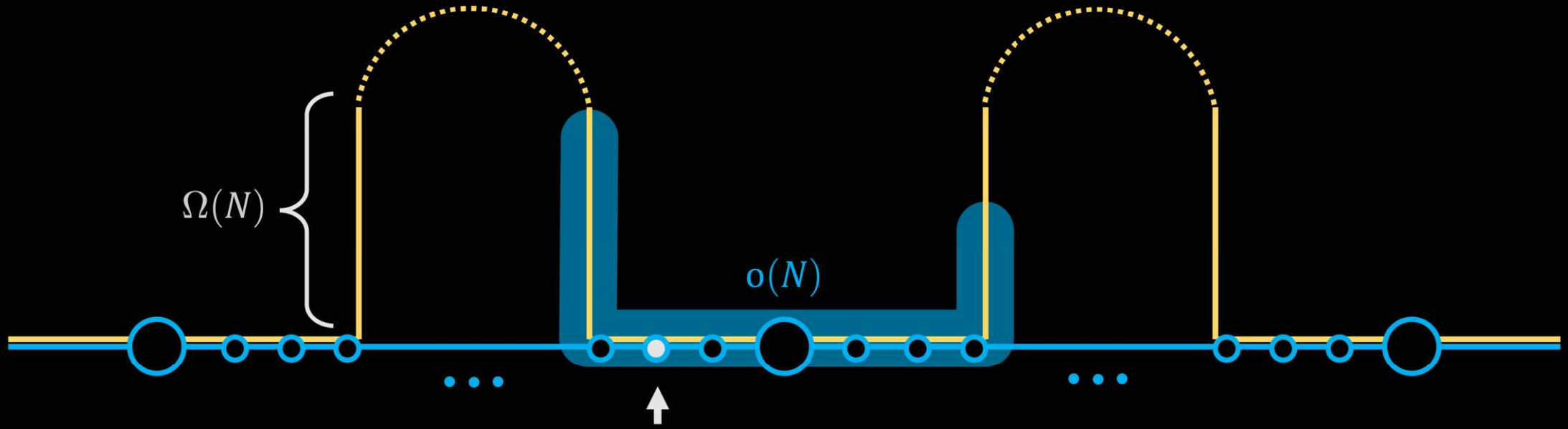
Gap result on paths in online-LOCAL

- Simulate sublinear-radius algorithm on the new path



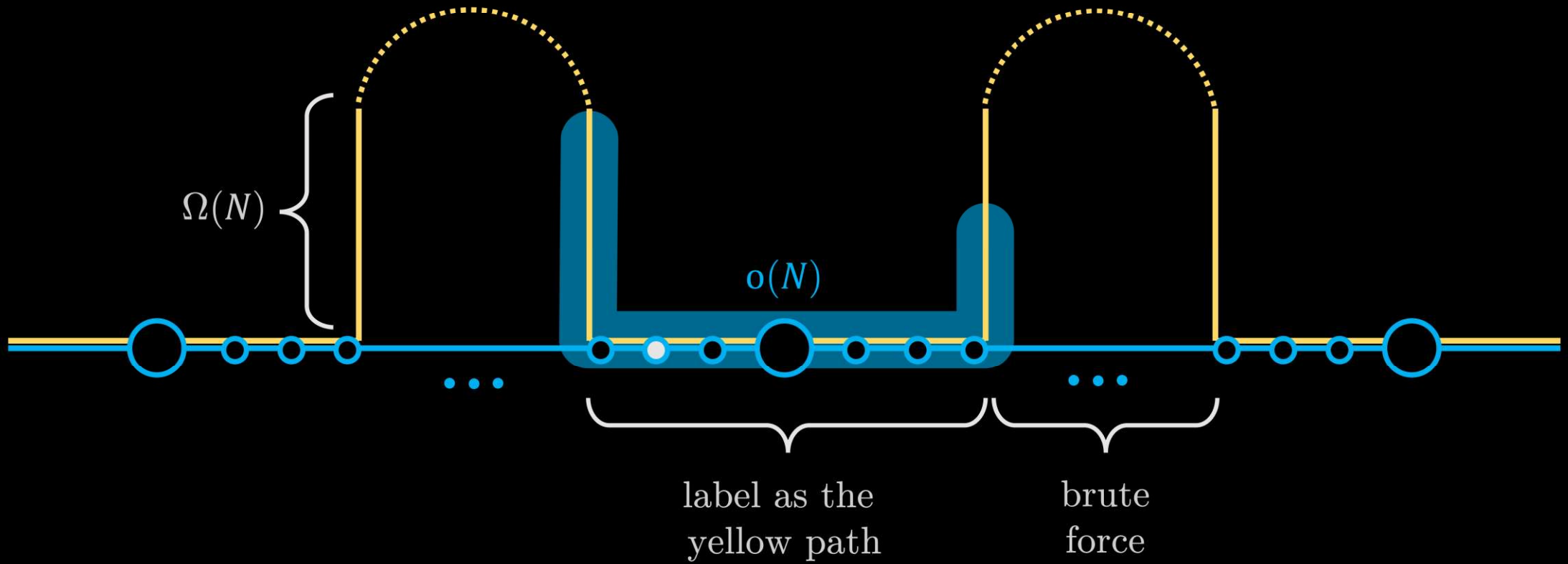
Gap result on paths in online-LOCAL

- Simulate algorithm on the new path

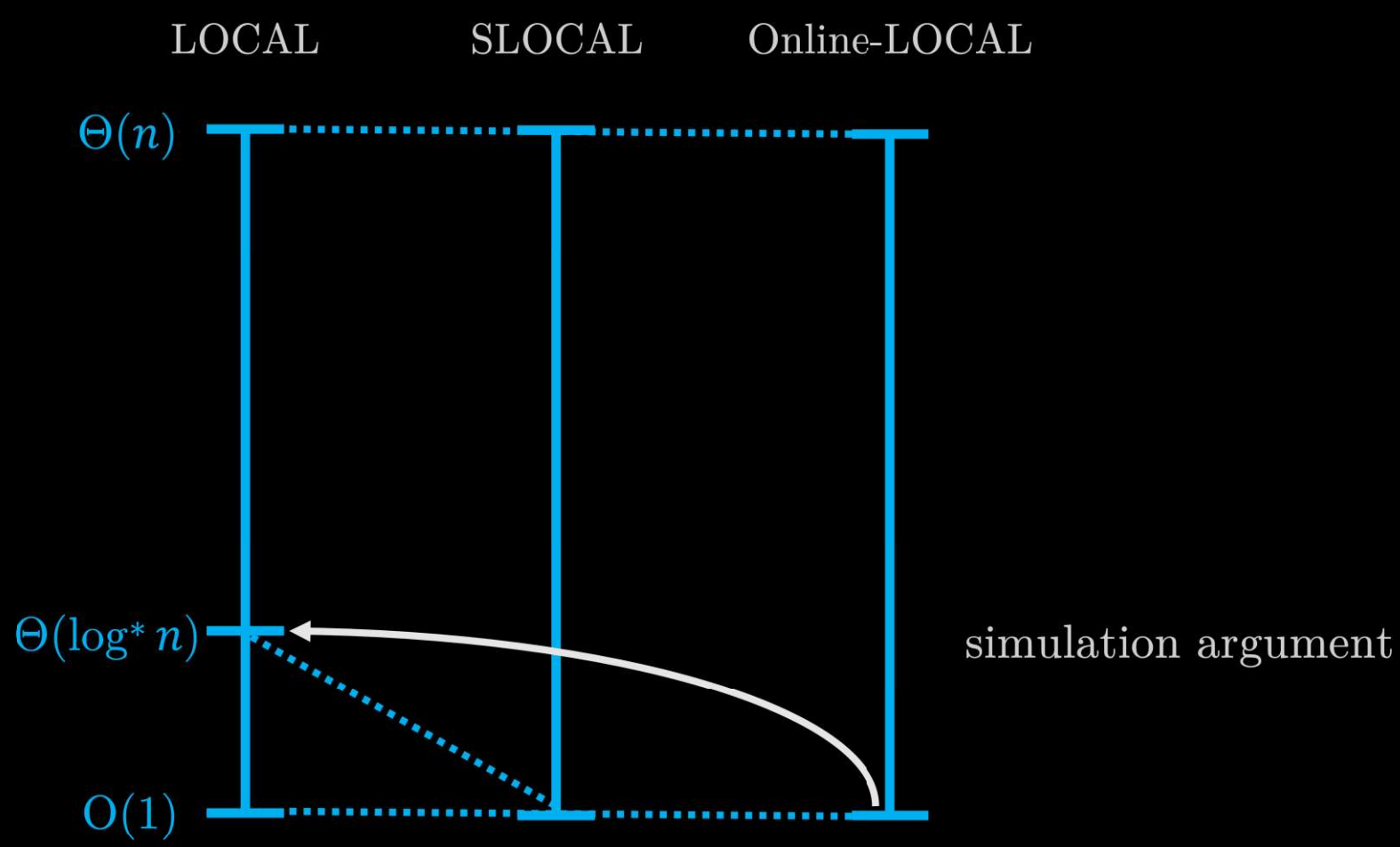


Gap result on paths in online-LOCAL

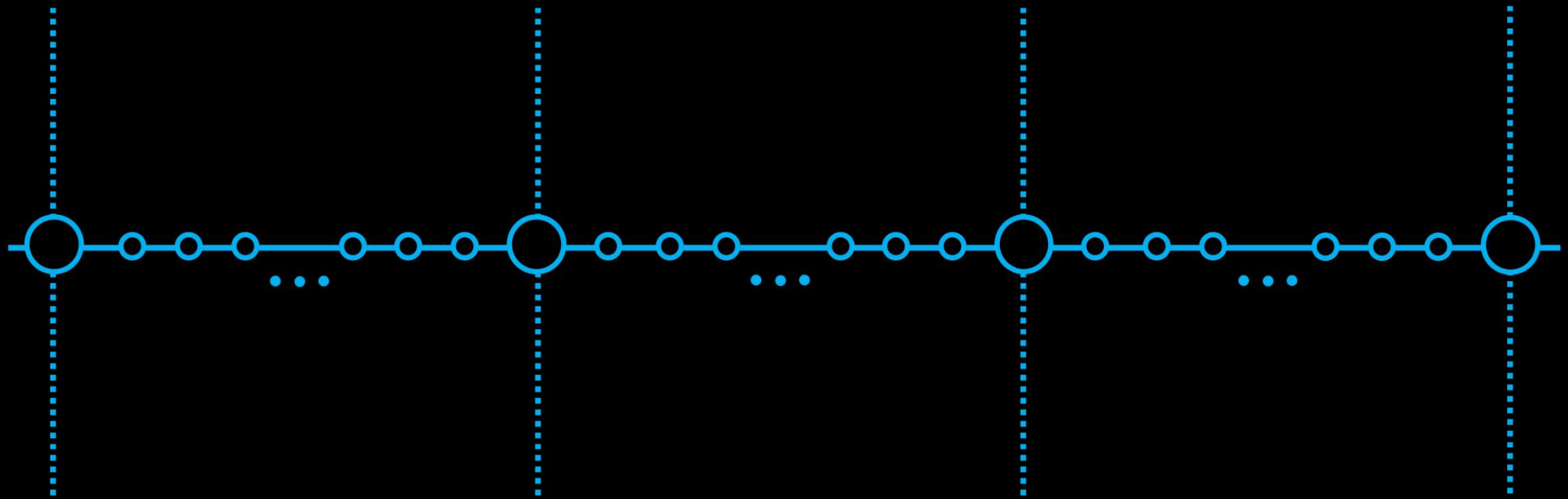
- Simulate algorithm on the new path



LCLs in paths and cycles

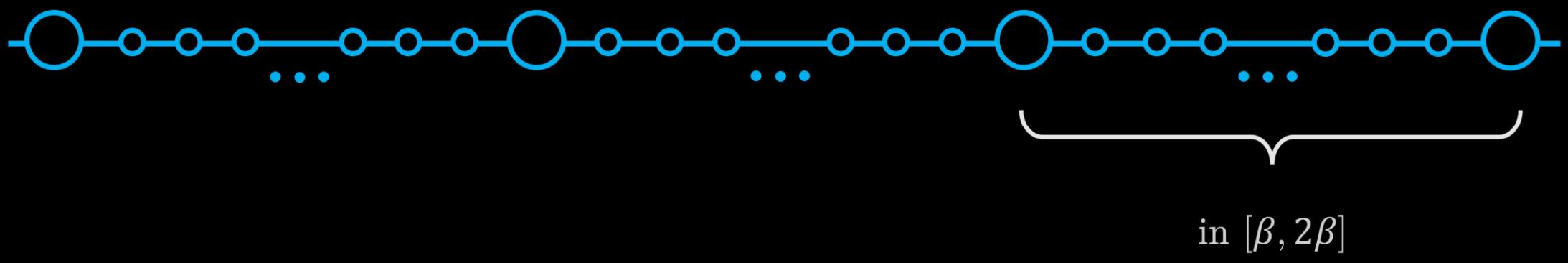


From constant-radius online-LOCAL to $O(\log^* n)$ LOCAL



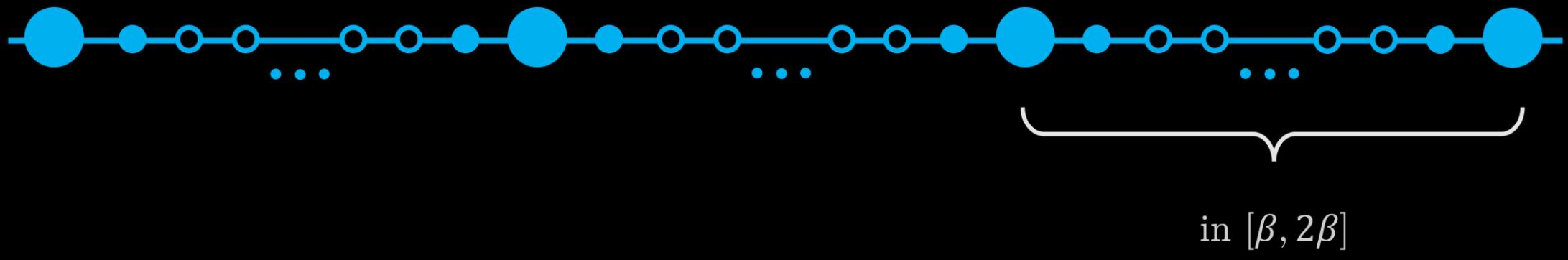
From constant-radius online-LOCAL to $O(\log^* n)$ LOCAL

- Construct distance-β coloring in $O(\log^* n)$ rounds
- Compute an (β, β)-ruling set



From constant-radius online-LOCAL to $O(\log^* n)$ LOCAL

- Construct distance-β coloring in $O(\log^* n)$ rounds
- Compute an (β, β)-ruling set
- Label radius-r neighborhood of each big node



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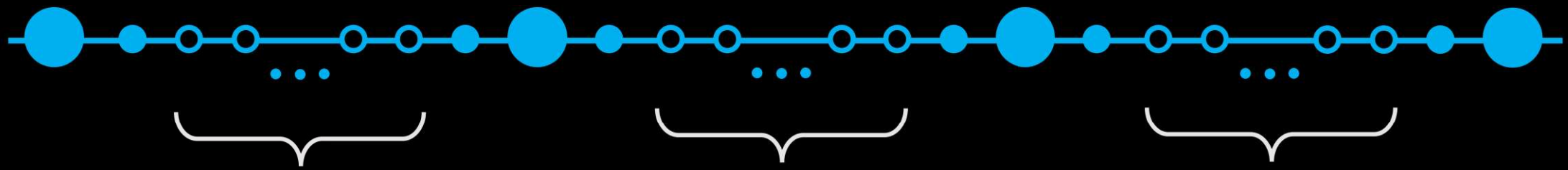
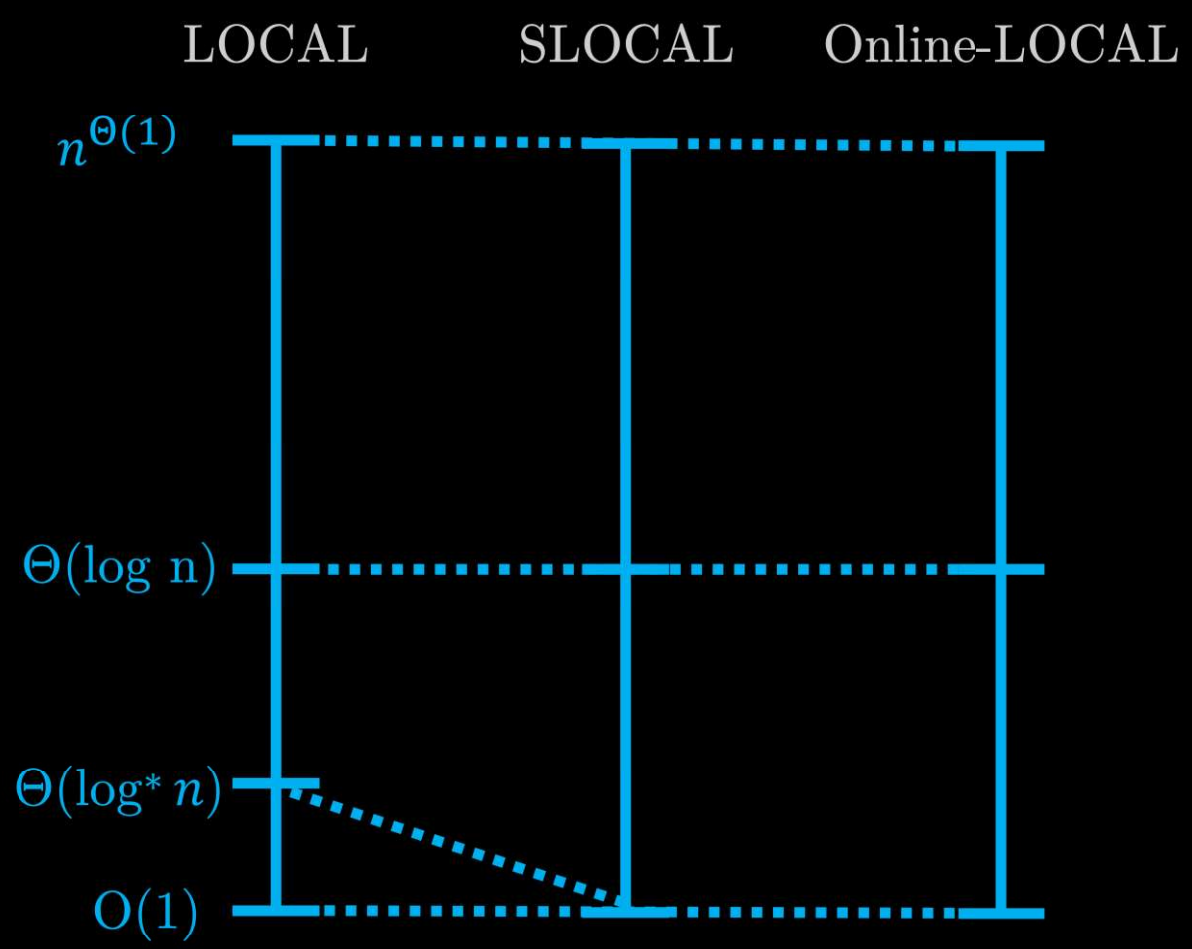
- Compute an (β, β)-ruling set
- Label radius-r neighborhood of each big node



Disjoint neighborhoods $\Longrightarrow$ use online-LOCAL algorithm for labeling

LCLs in rooted trees



LCLs on grids

LCLs in 2-D grids

	LOCAL	Online-LOCAL	references
2-coloring	$\Theta(\sqrt{n})$	$\Theta(\sqrt{n})$	easy
3-coloring	$\Theta(\sqrt{n})$	$O(\log n)$	Brandt et al., PODC'17
4-coloring	$\Theta(\log^* n)$	$O(1)$	Brandt et al., PODC'17
5-coloring	$\Theta(\log^* n)$	$O(1)$	Brandt et al., PODC'17

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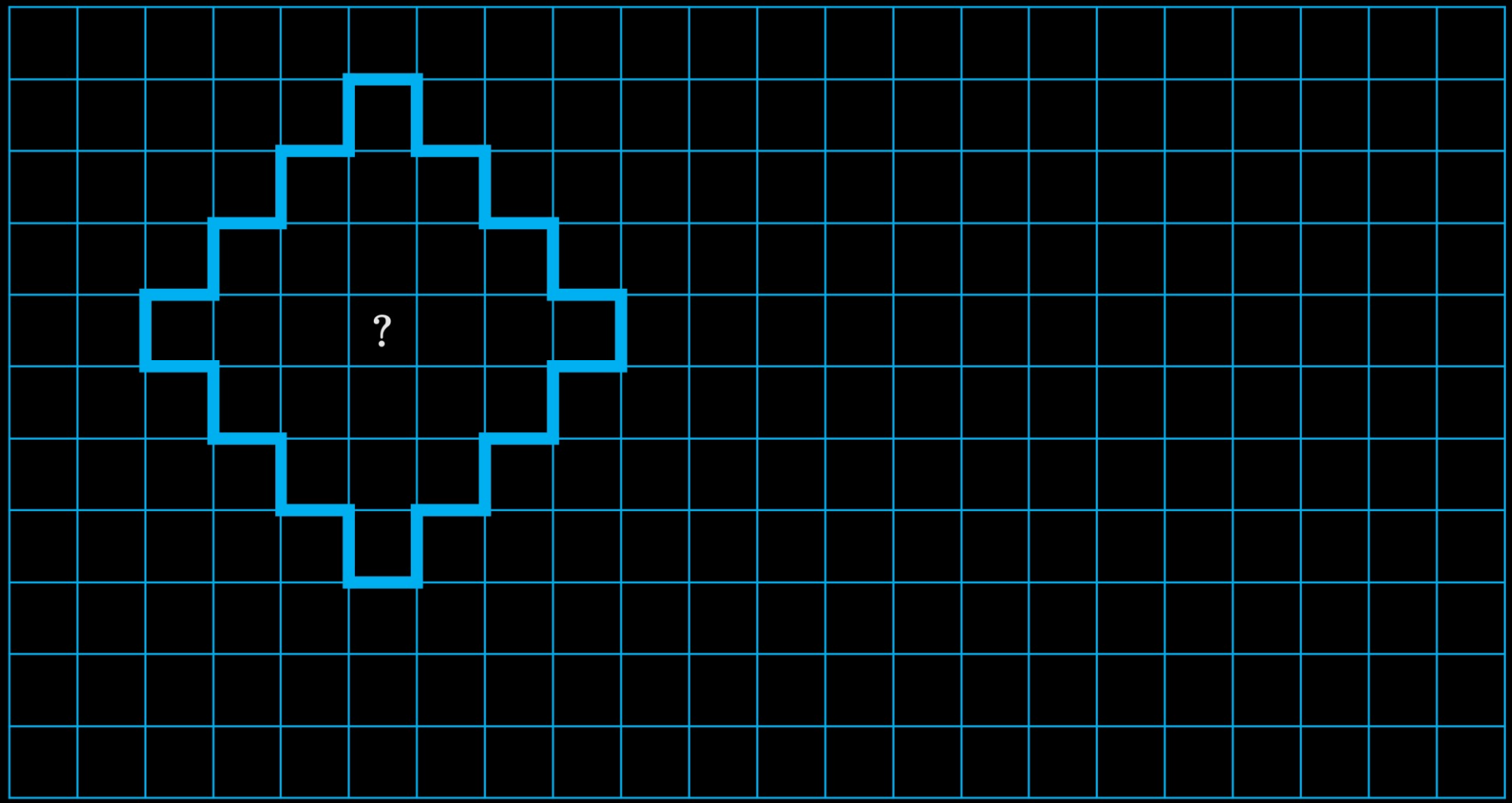
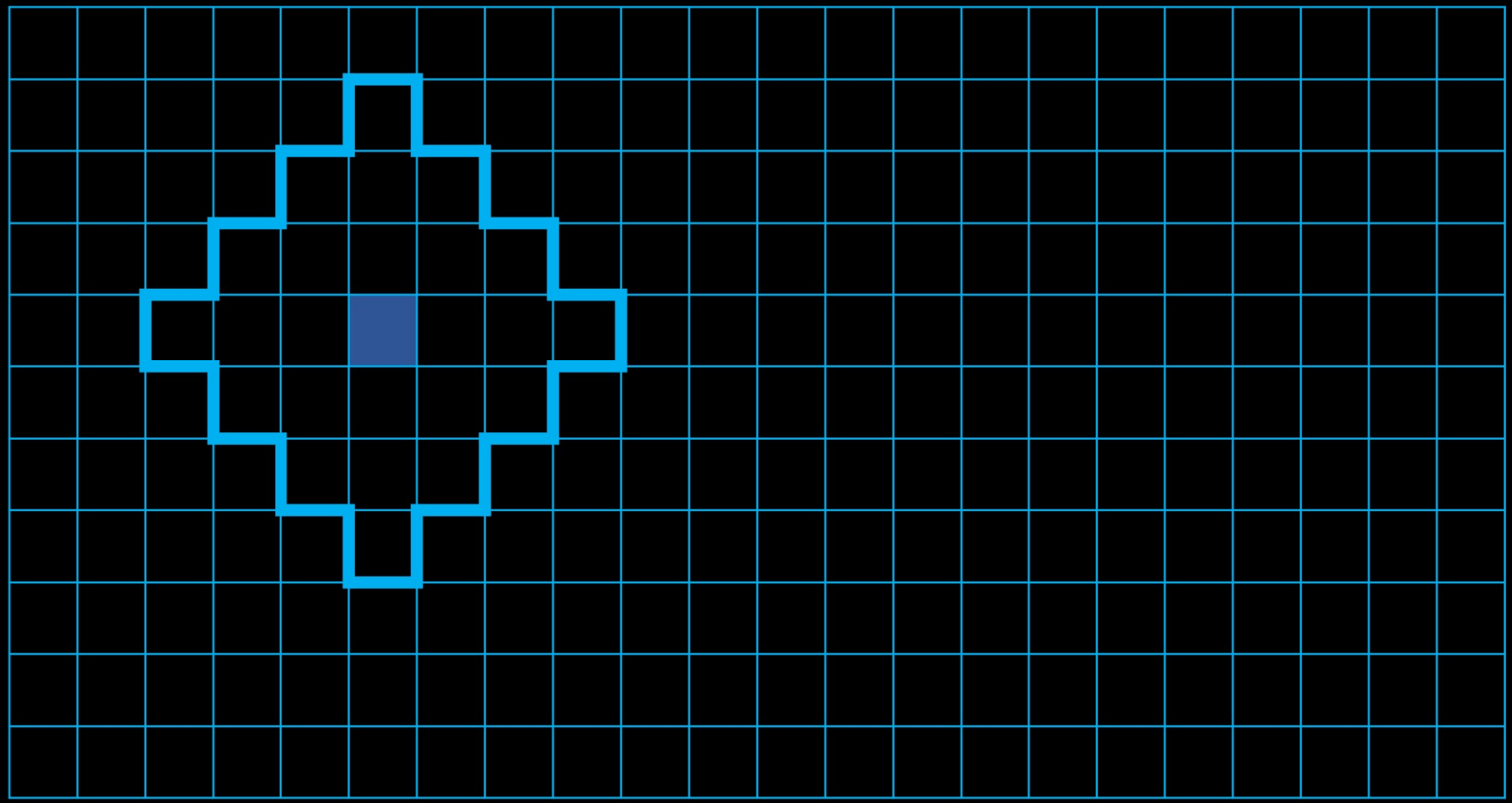
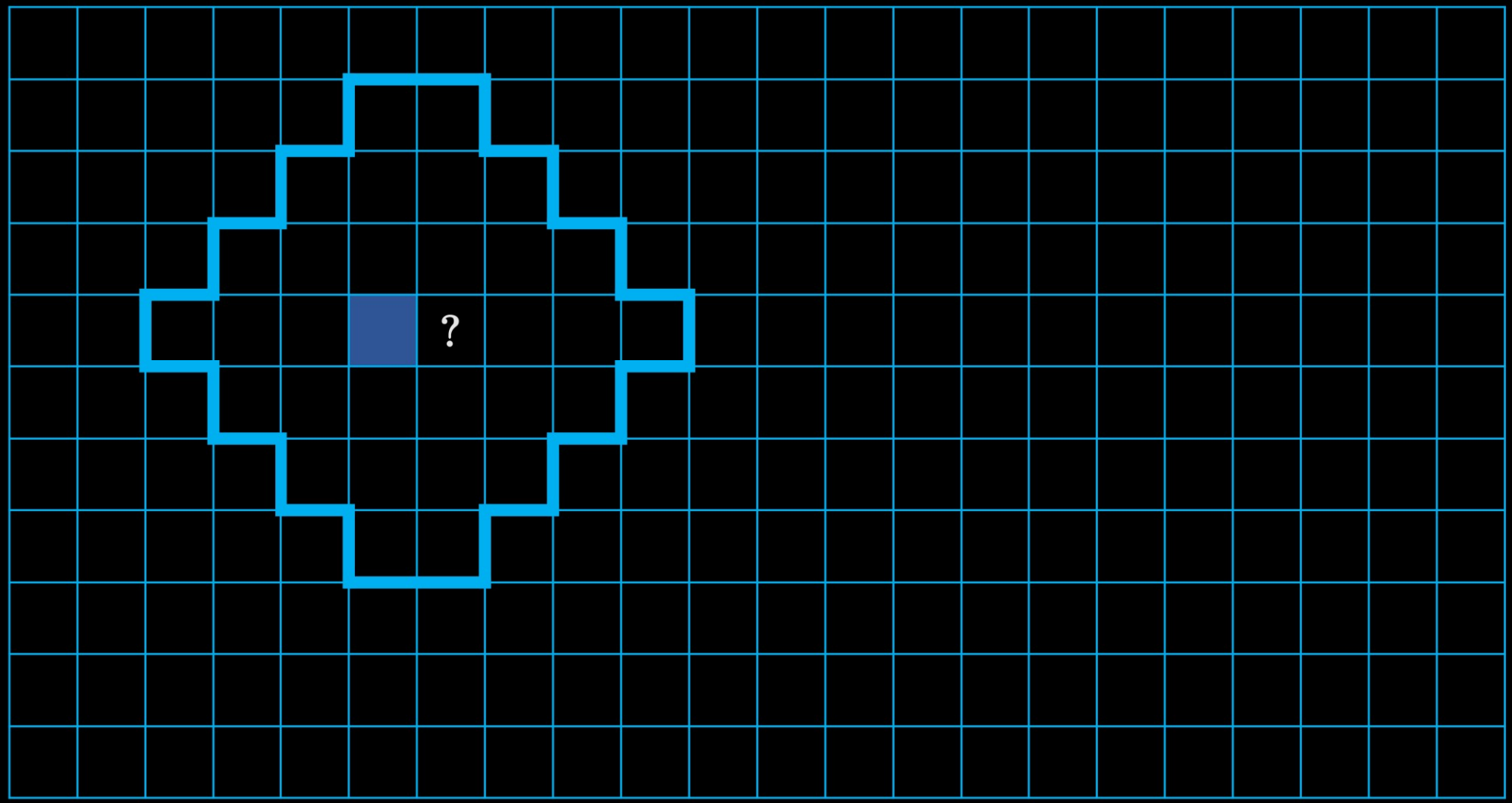
3-Coloring Grids



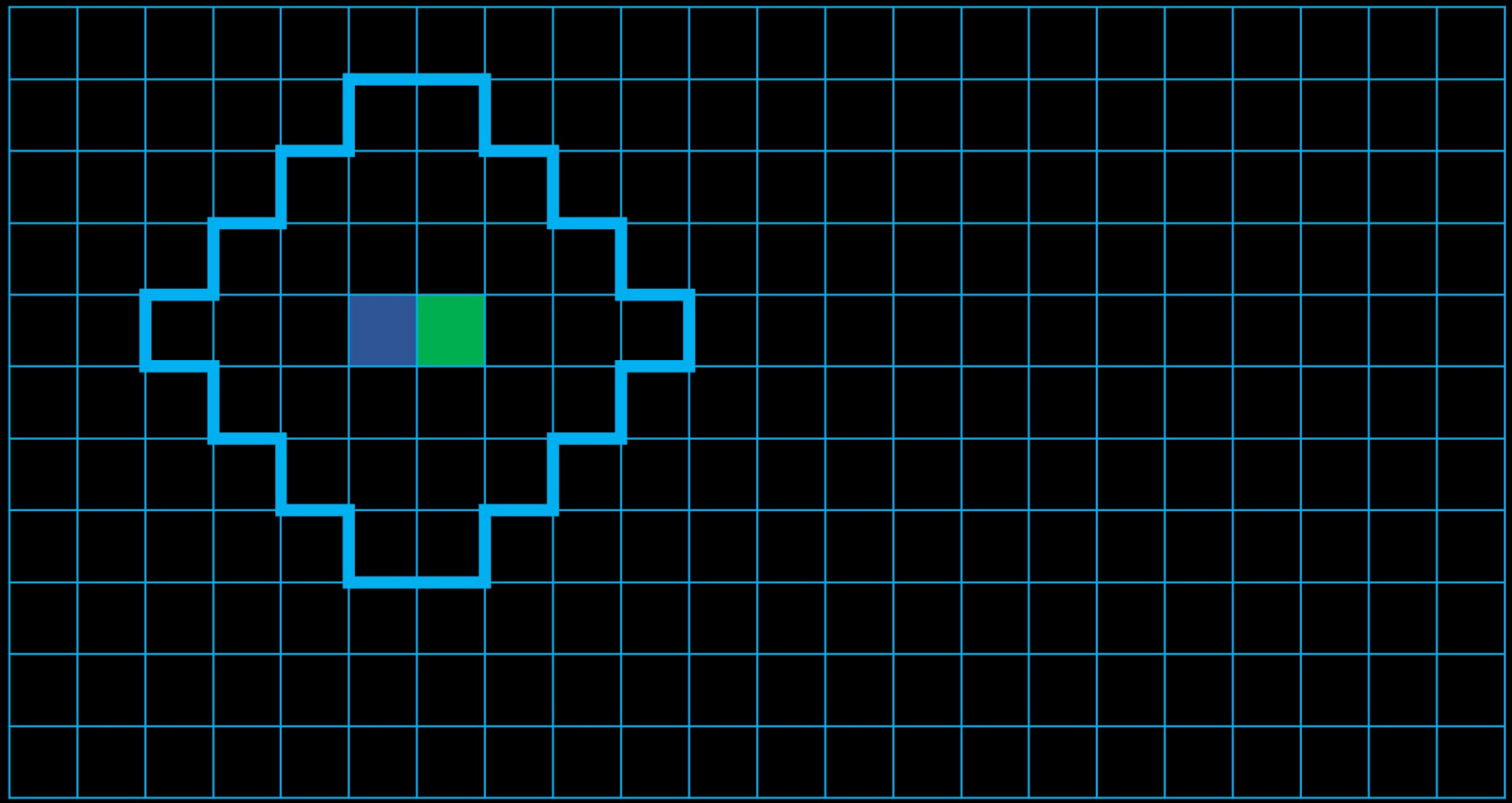
3-Coloring Grids



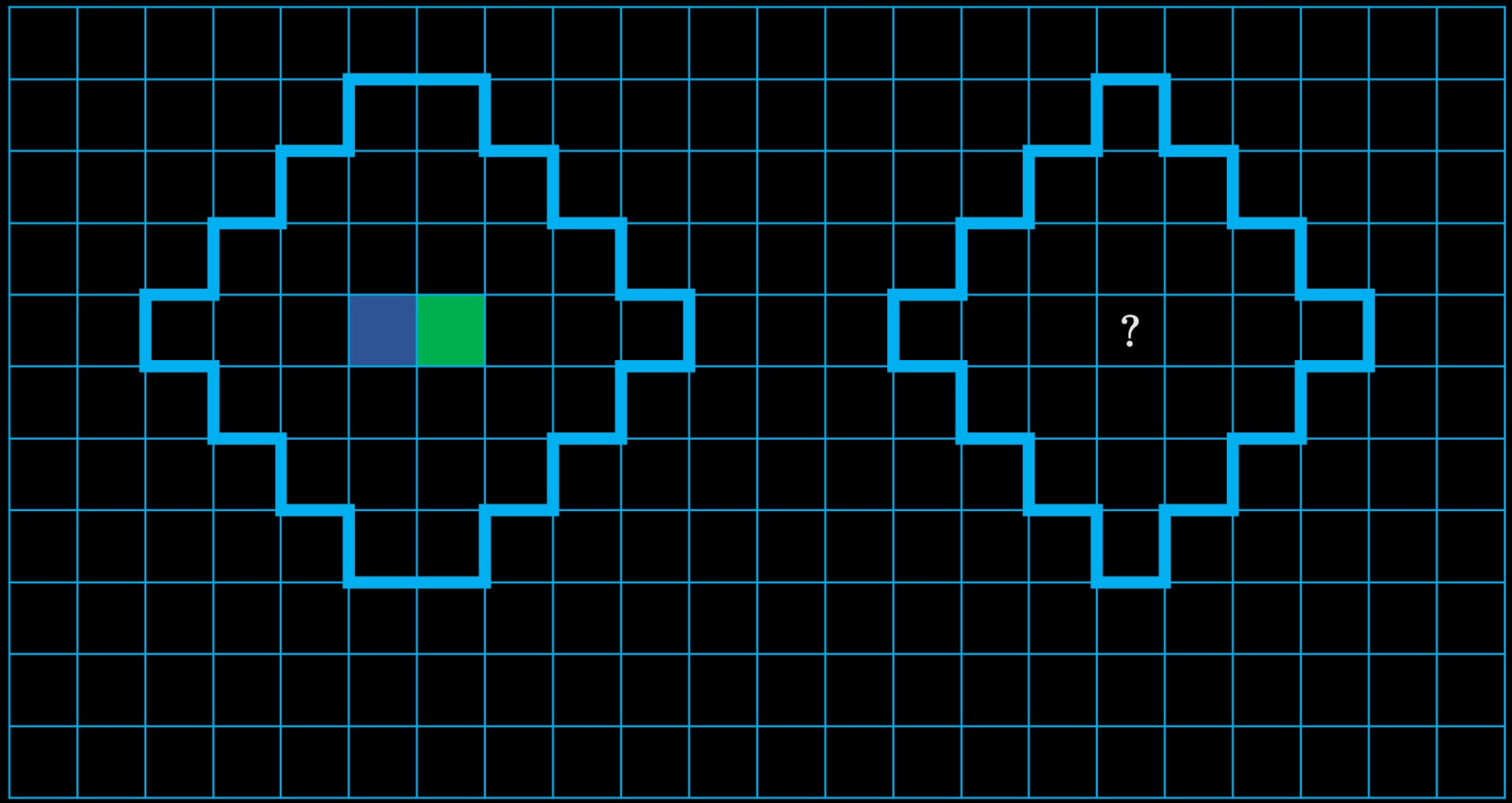
3-Coloring Grids



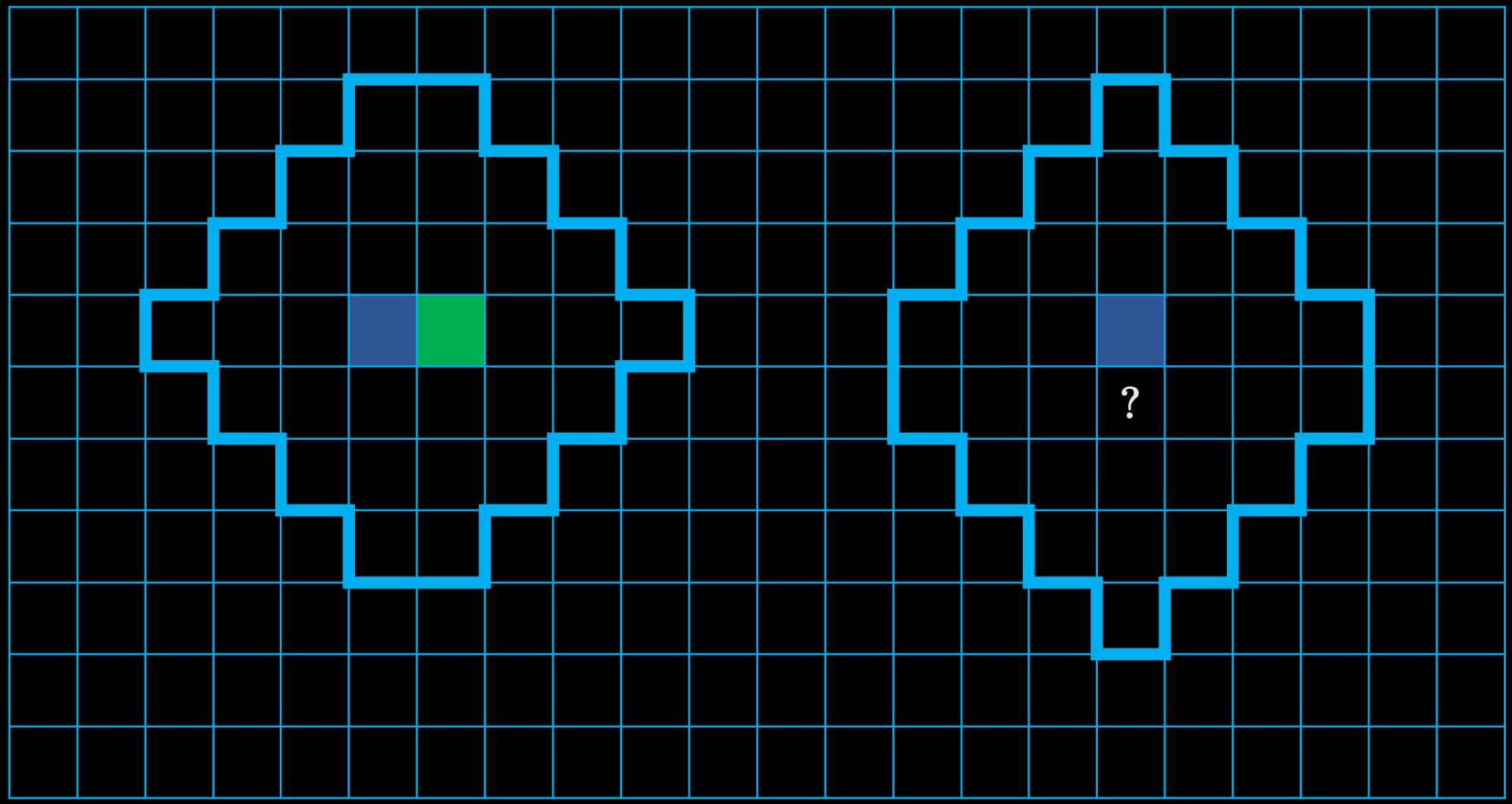
3-Coloring Grids



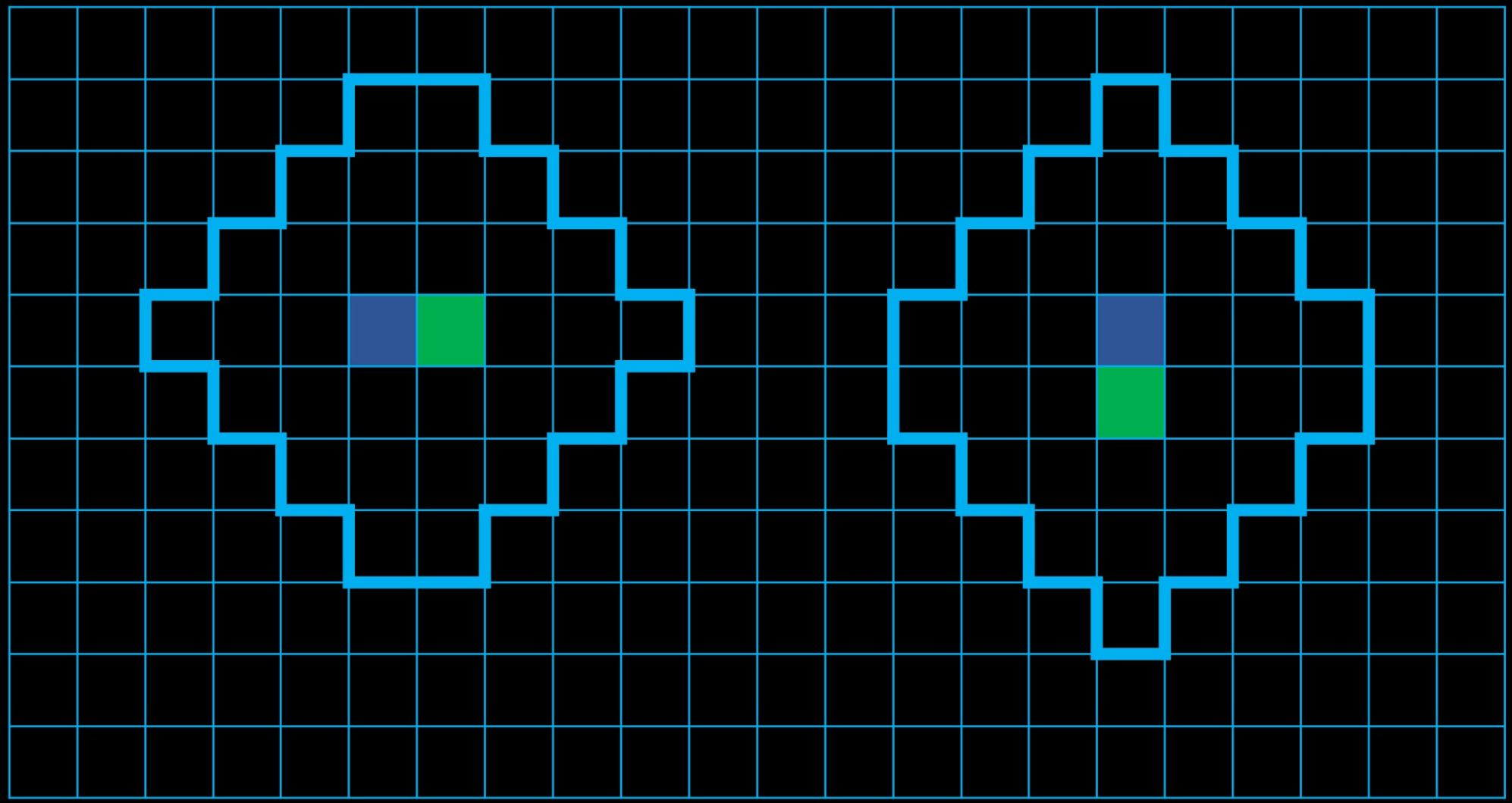
3-Coloring Grids



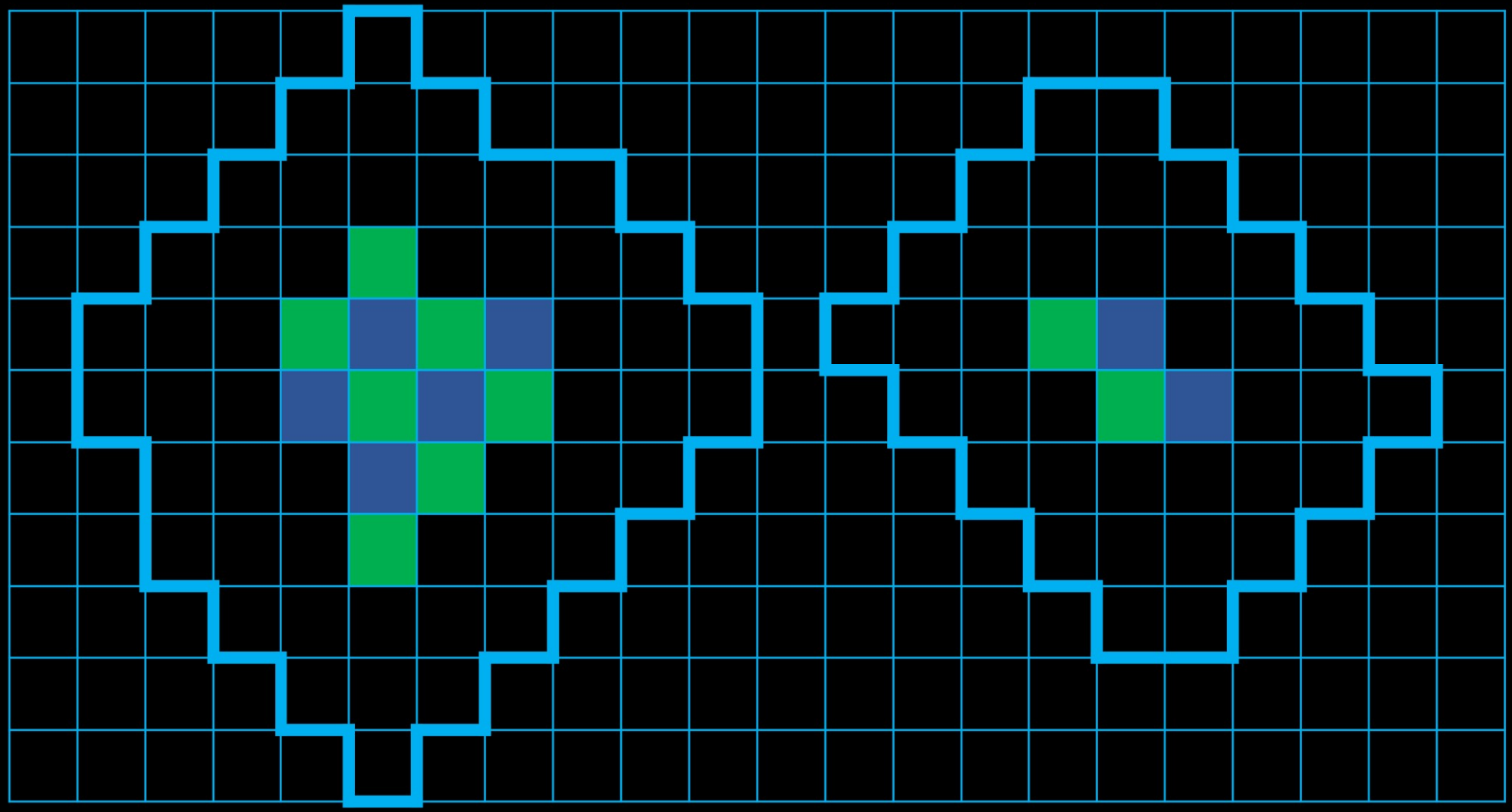
3-Coloring Grids



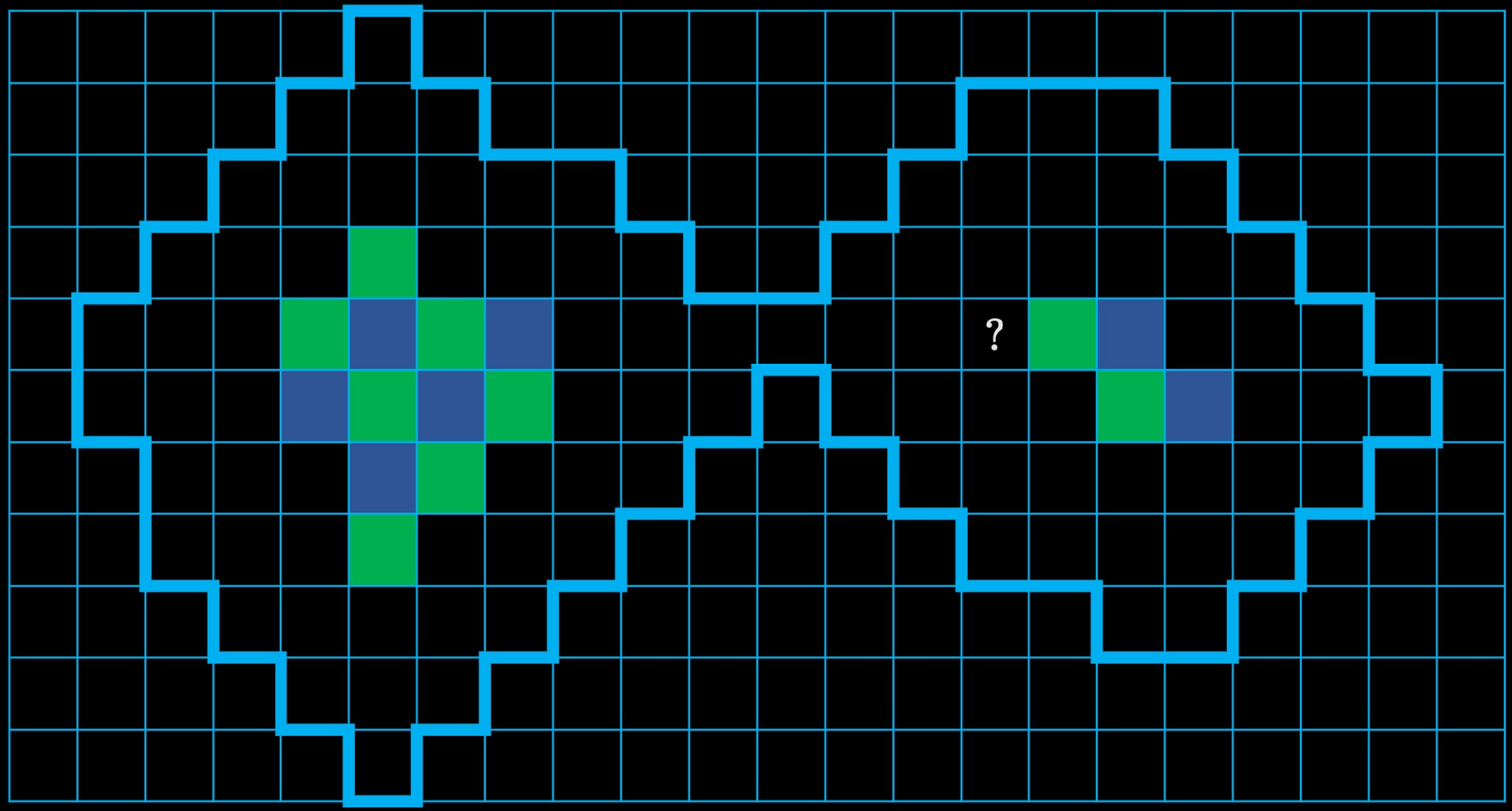
3-Coloring Grids



3-Coloring Grids

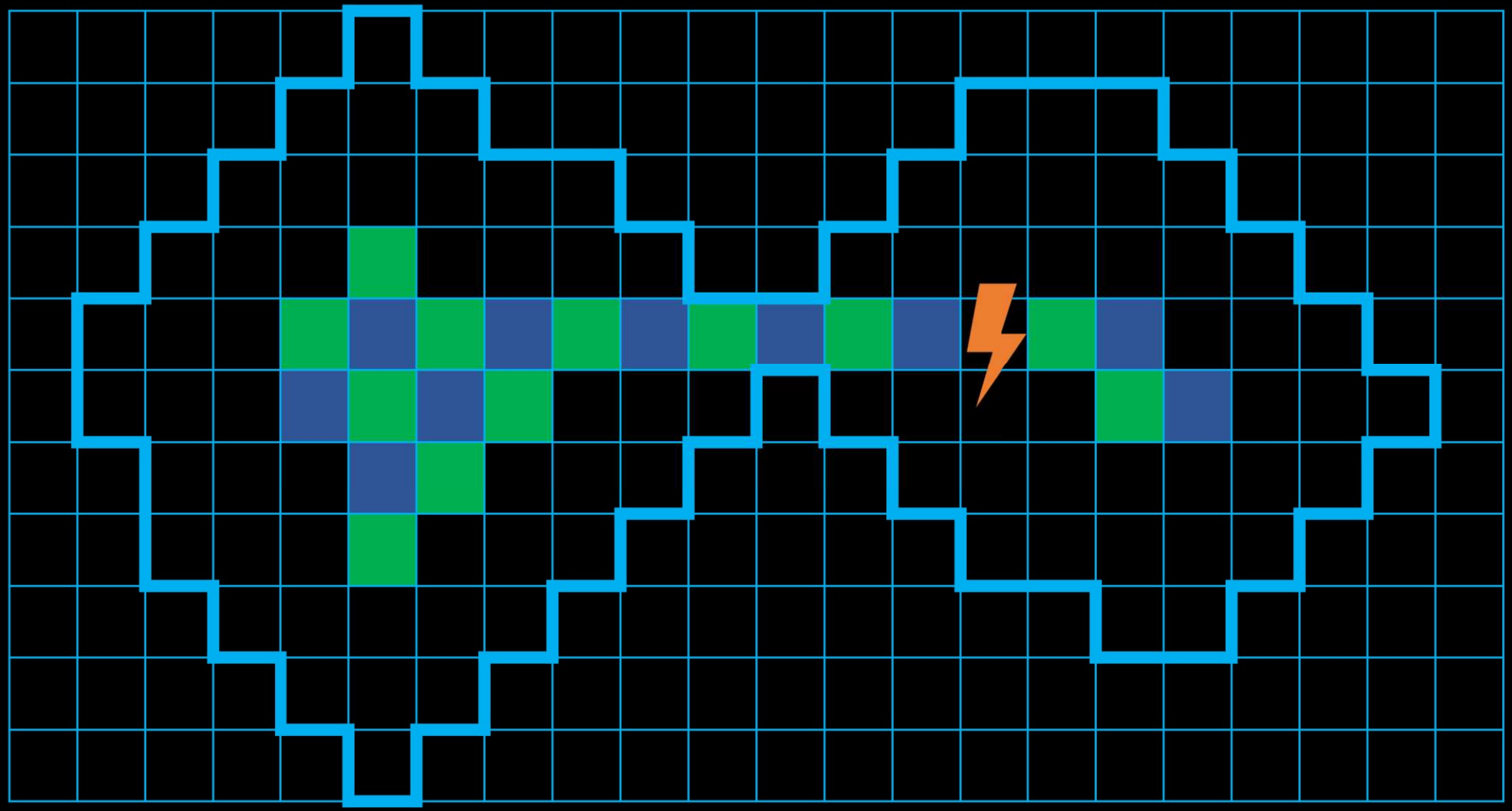


3-Coloring Grids



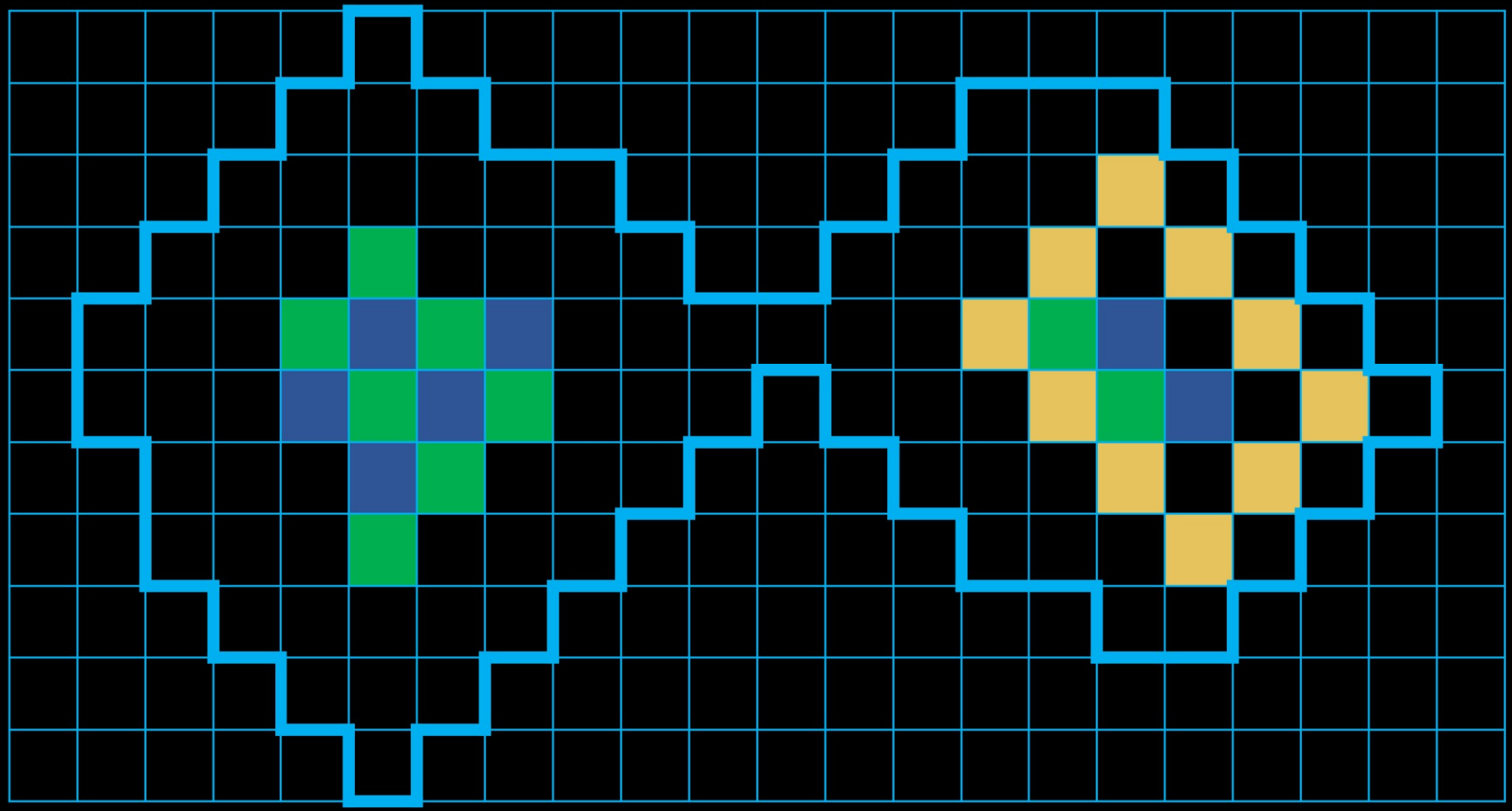
3-Coloring Grids

- Parities do not match



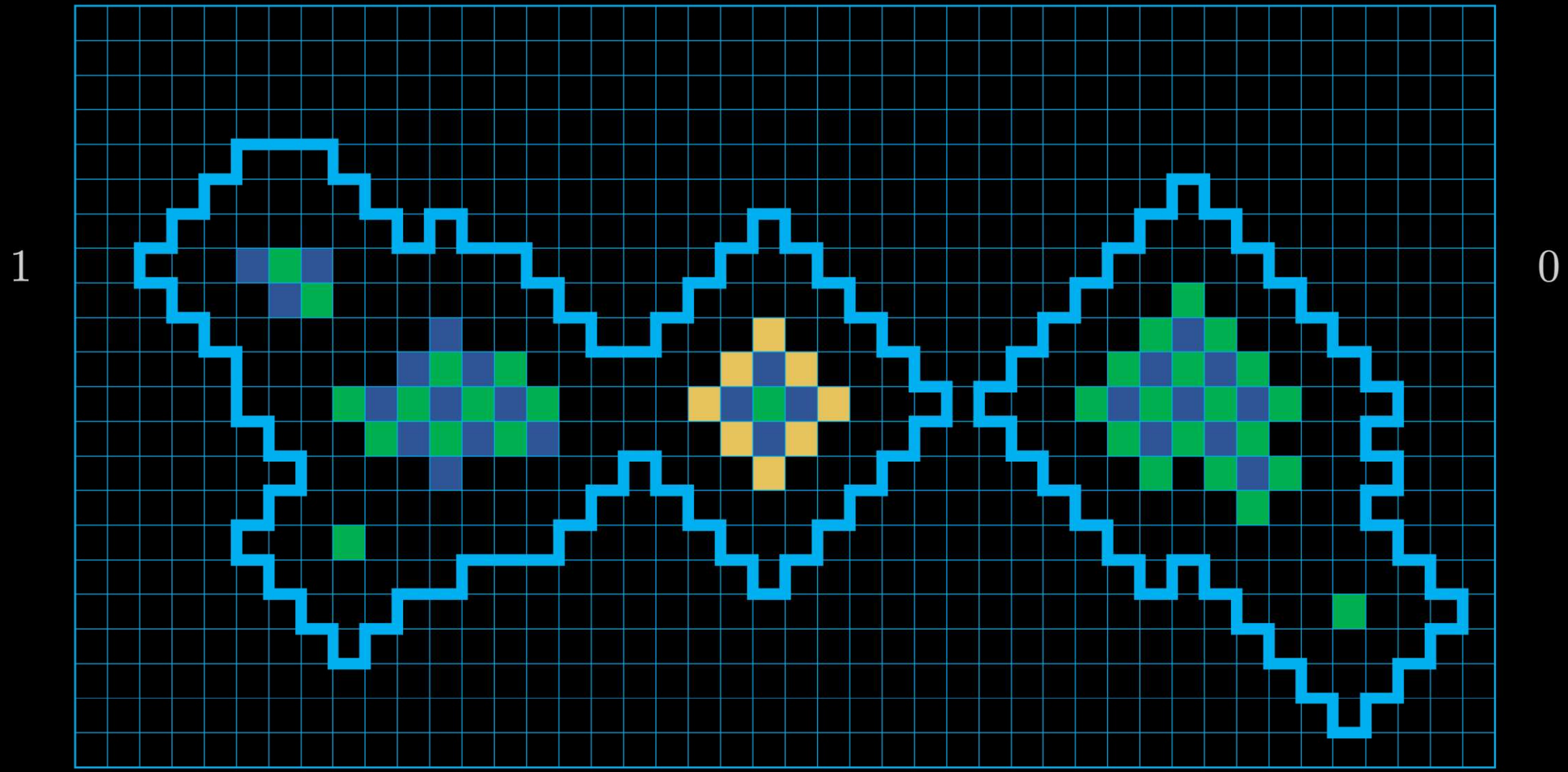
3-Coloring Grids

- Switching parity



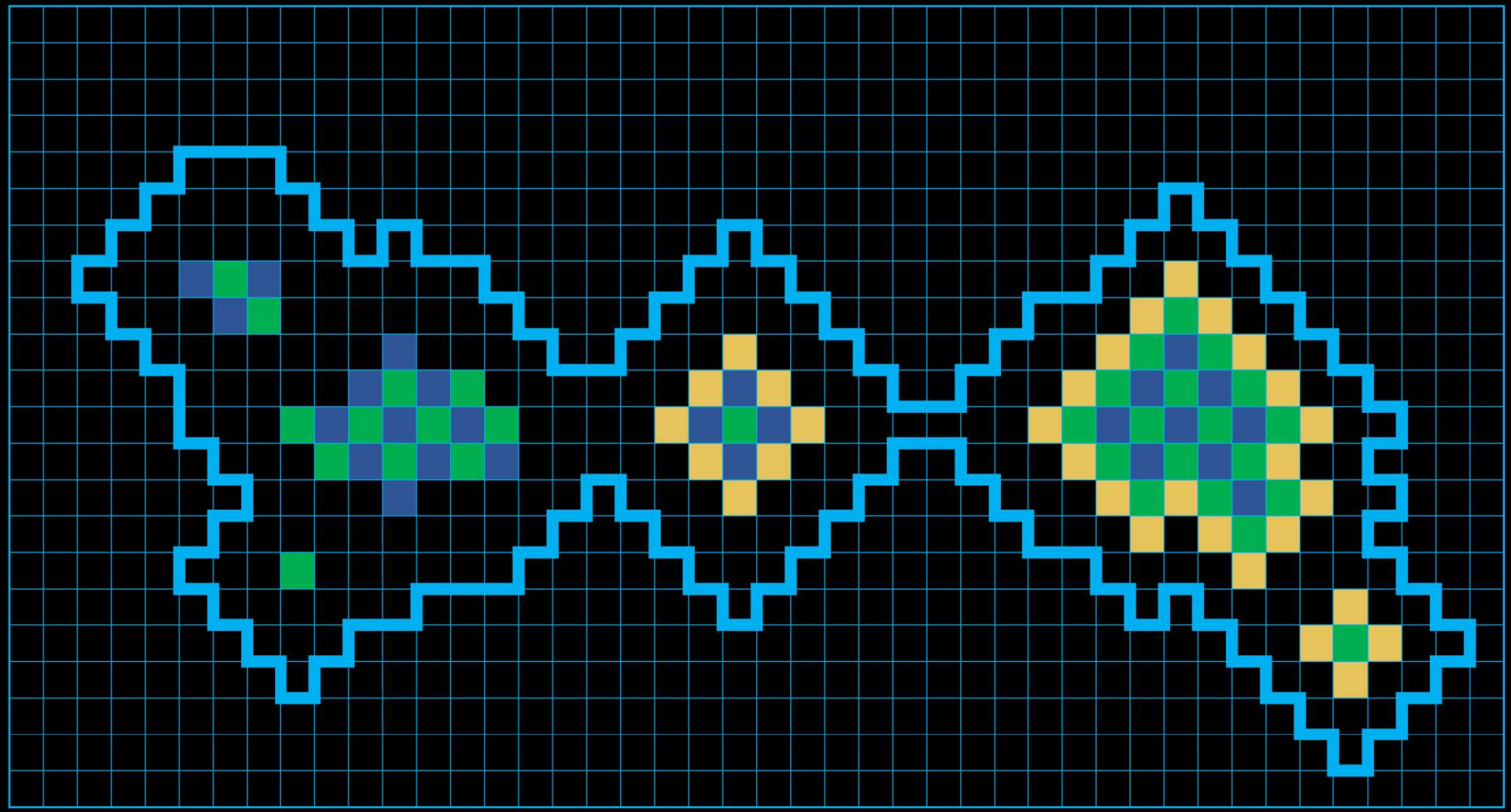
3-Coloring Grids

- Add boundary count



3-Coloring Grids

1



Open Questions

- Lower bound for 3-coloring 2-D grids in online-LOCAL?
- Or better upper bound?
- Complexity 3-coloring in 2-D grids in the SLOCAL model?
- Lower bound for 3-coloring unrooted trees in SLOCAL?
- Same in online-LOCAL?