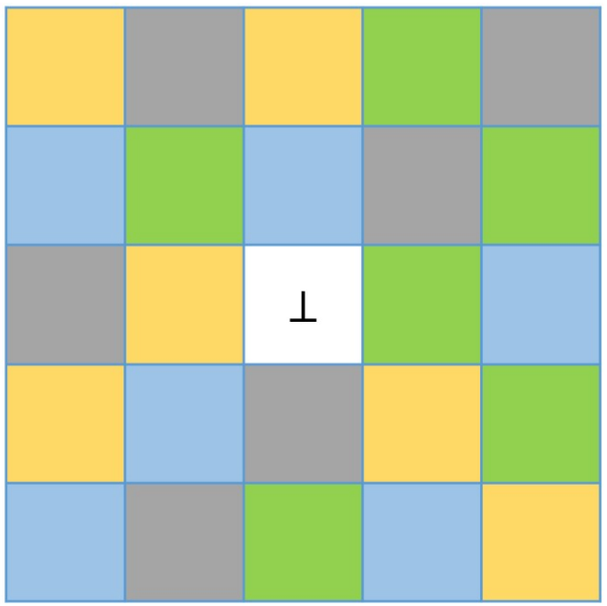
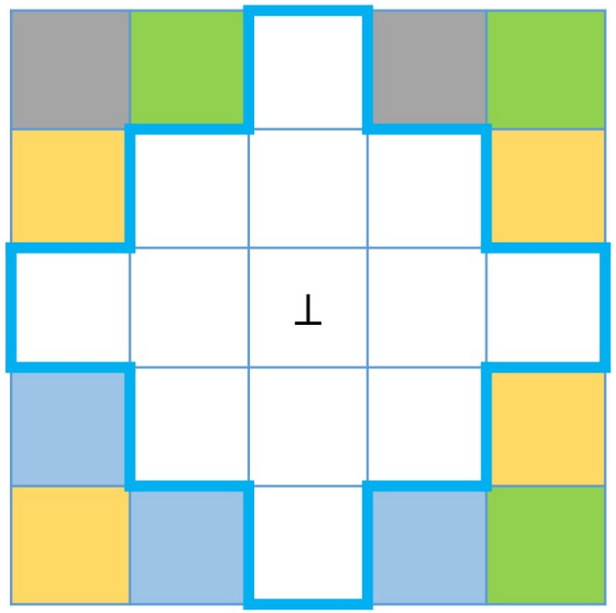


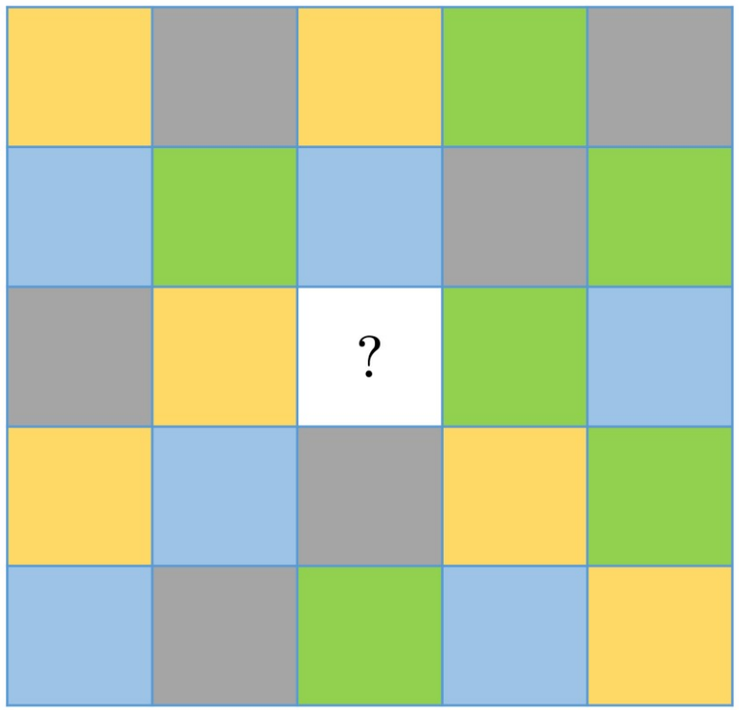
Local Mending

Darya Melnyk
Aalto University
SIROCCO 2022



Joint work with Alkida Balliu, Juho Hirvonen, Dennis Olivetti, Joel Rybicki, Jukka Suomela

4-Coloring Grids



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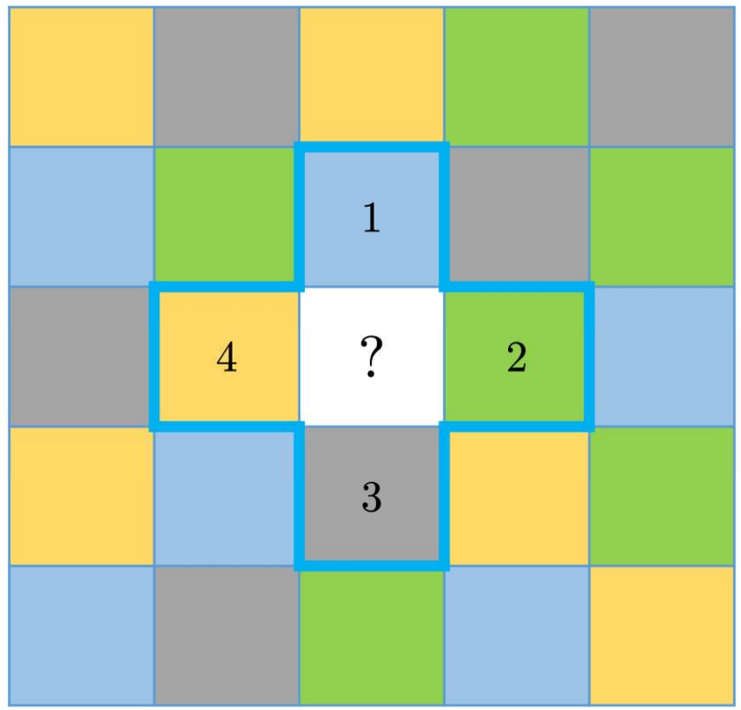

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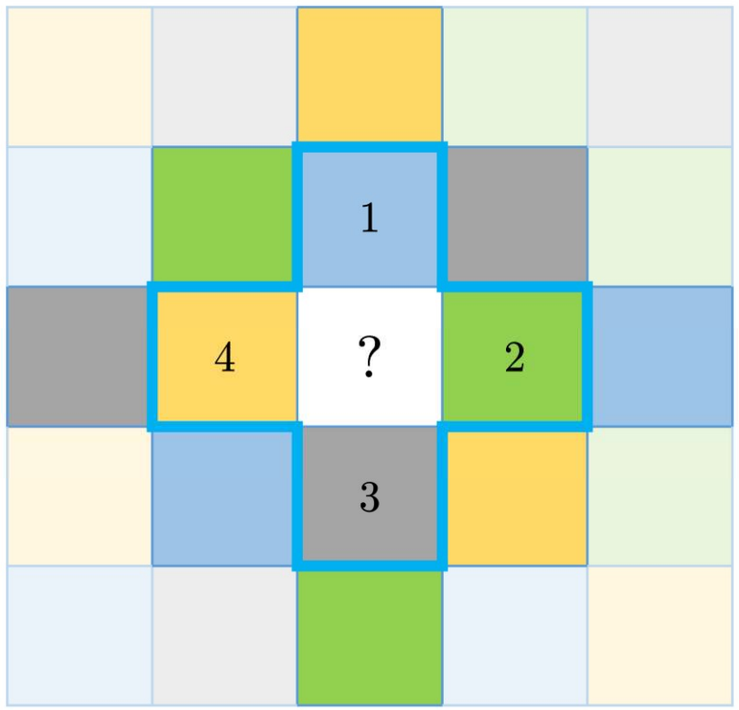

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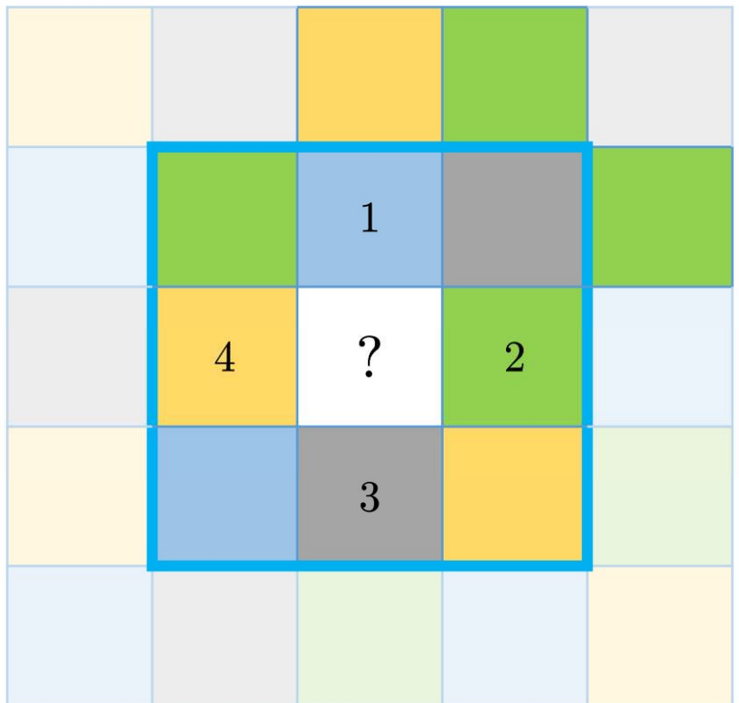

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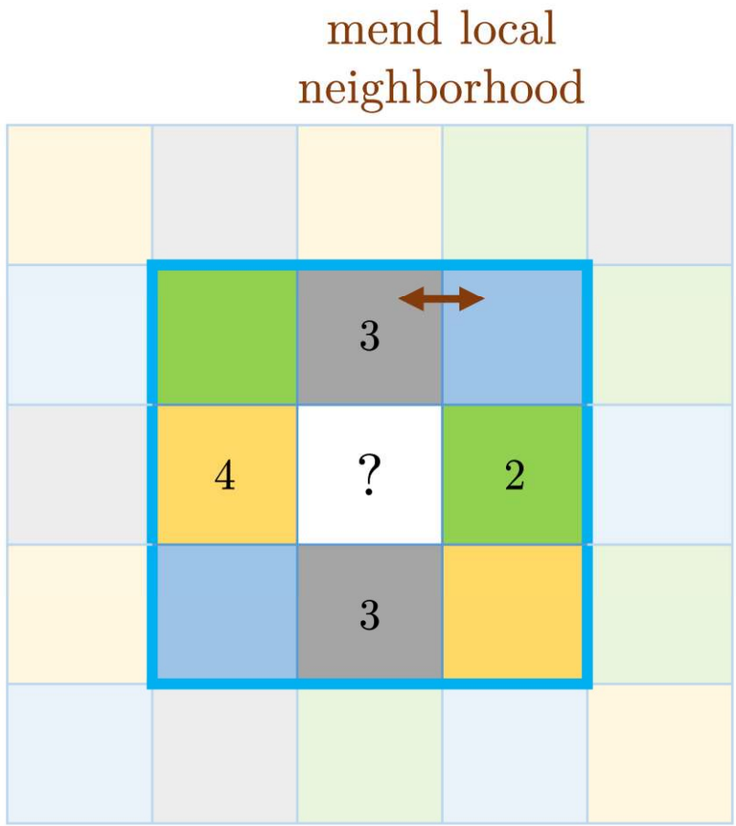
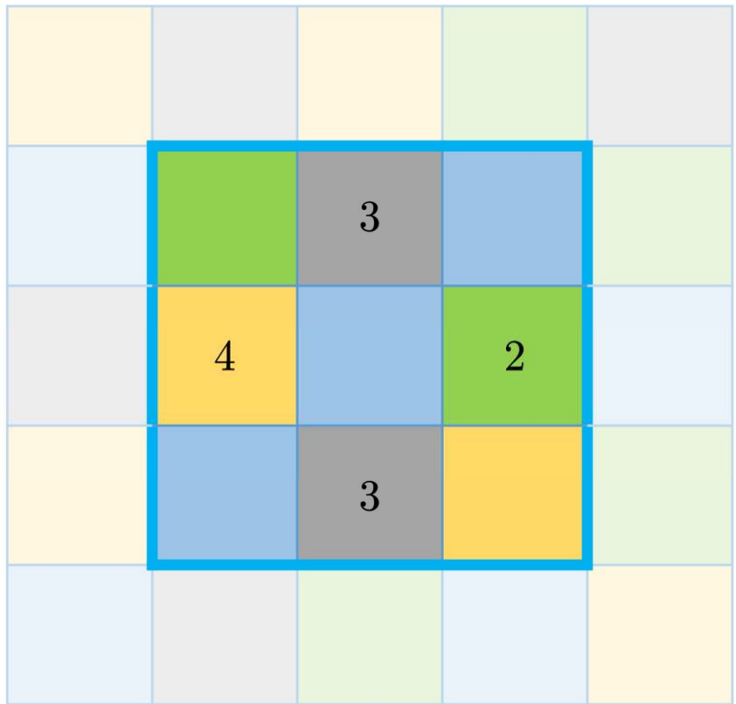
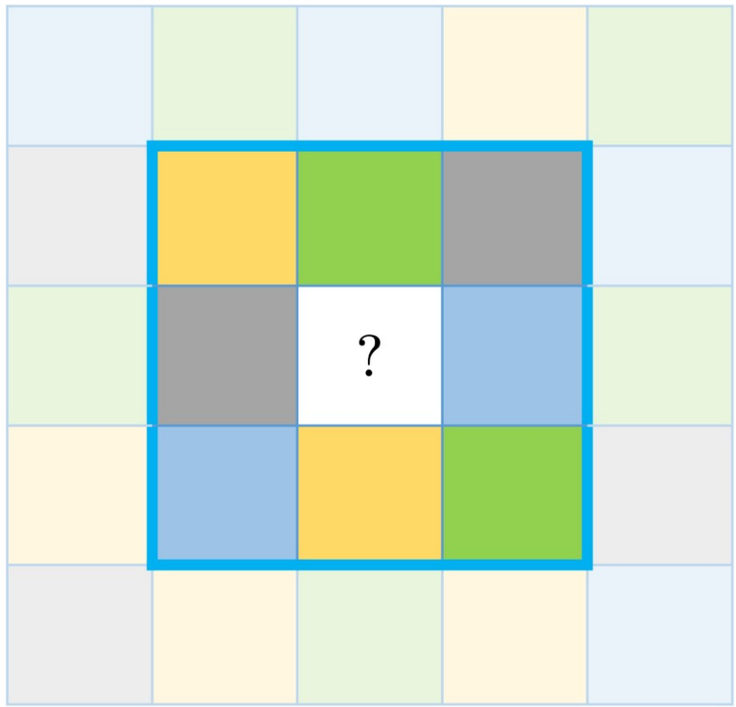


4-Coloring Grids



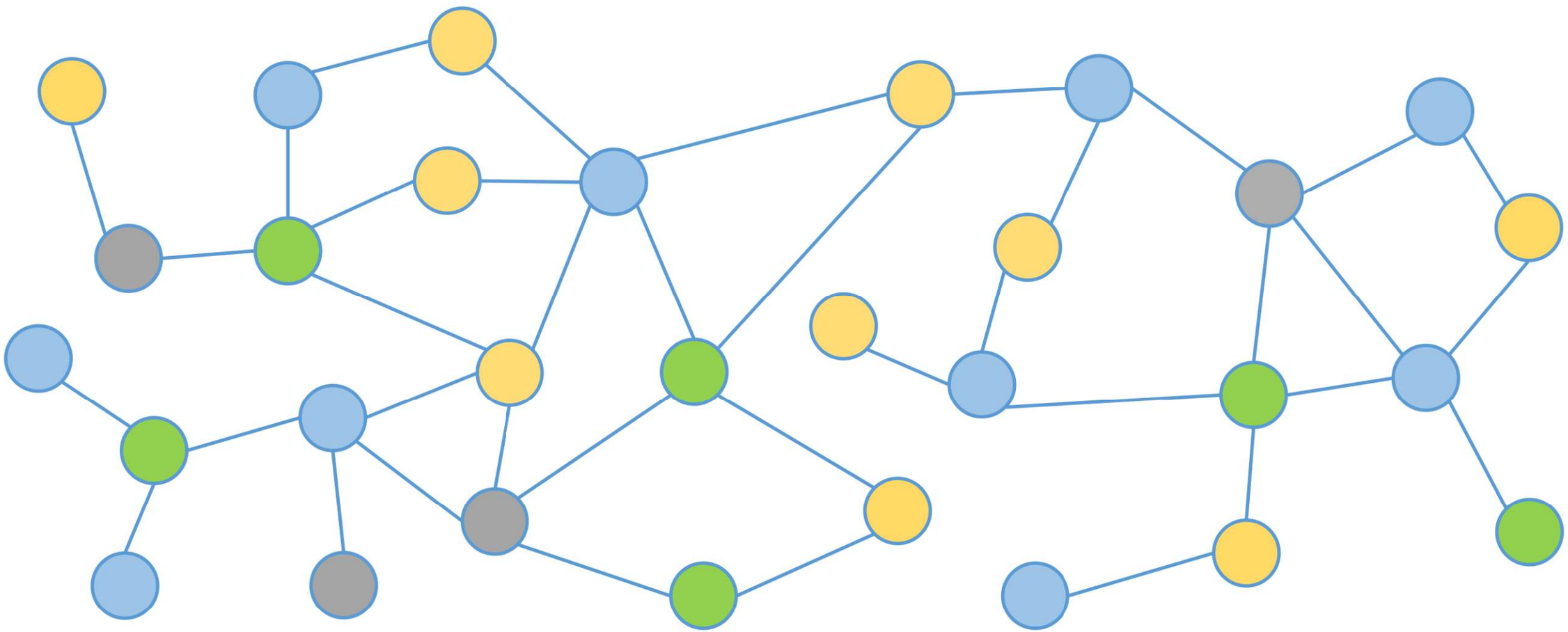
4-Coloring Grids



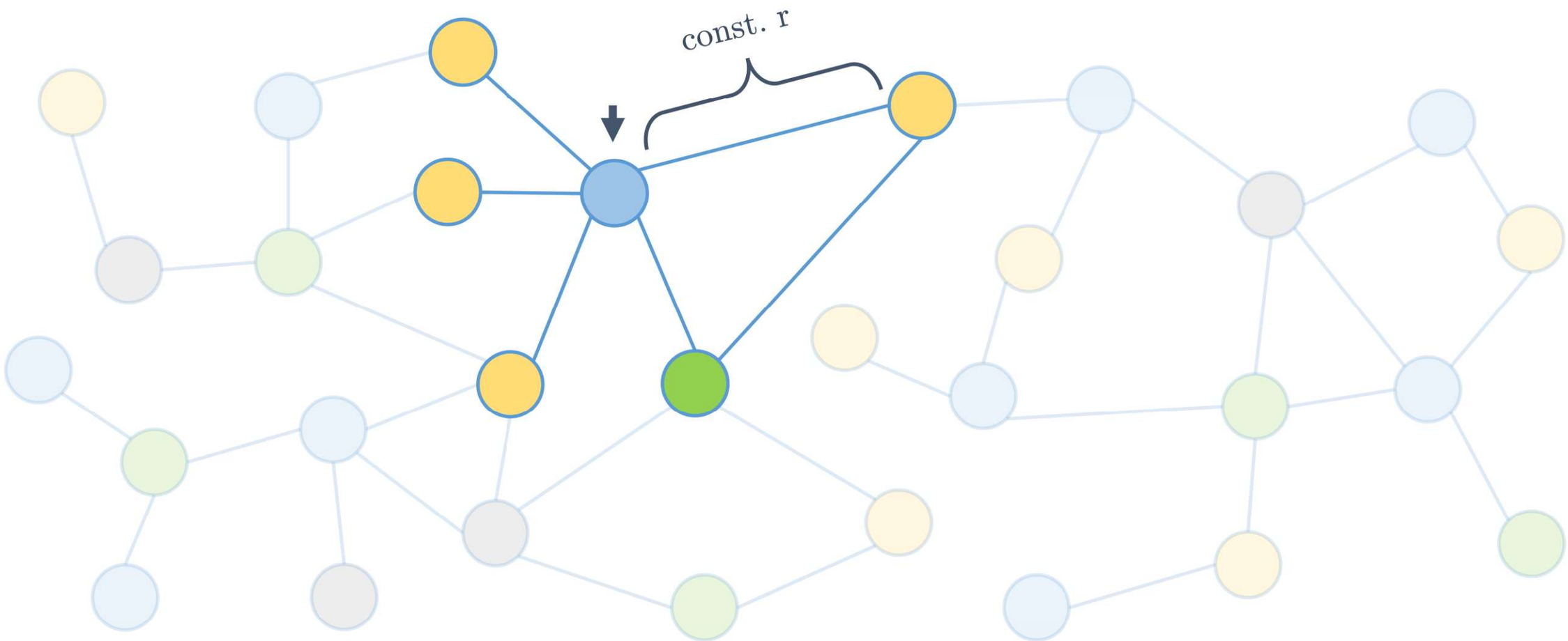
Any conflict can be fixed this way!

How to formalize mending?

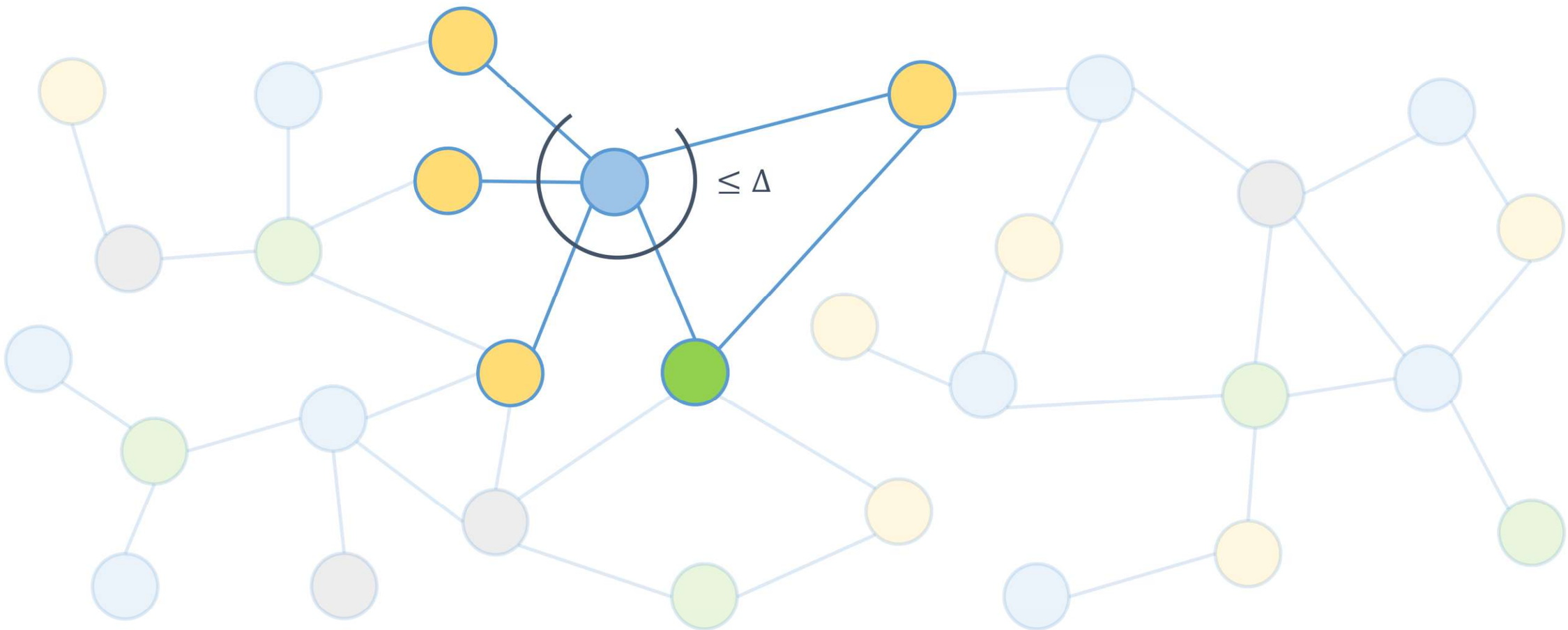
LCLs – locally checkable labeling problems



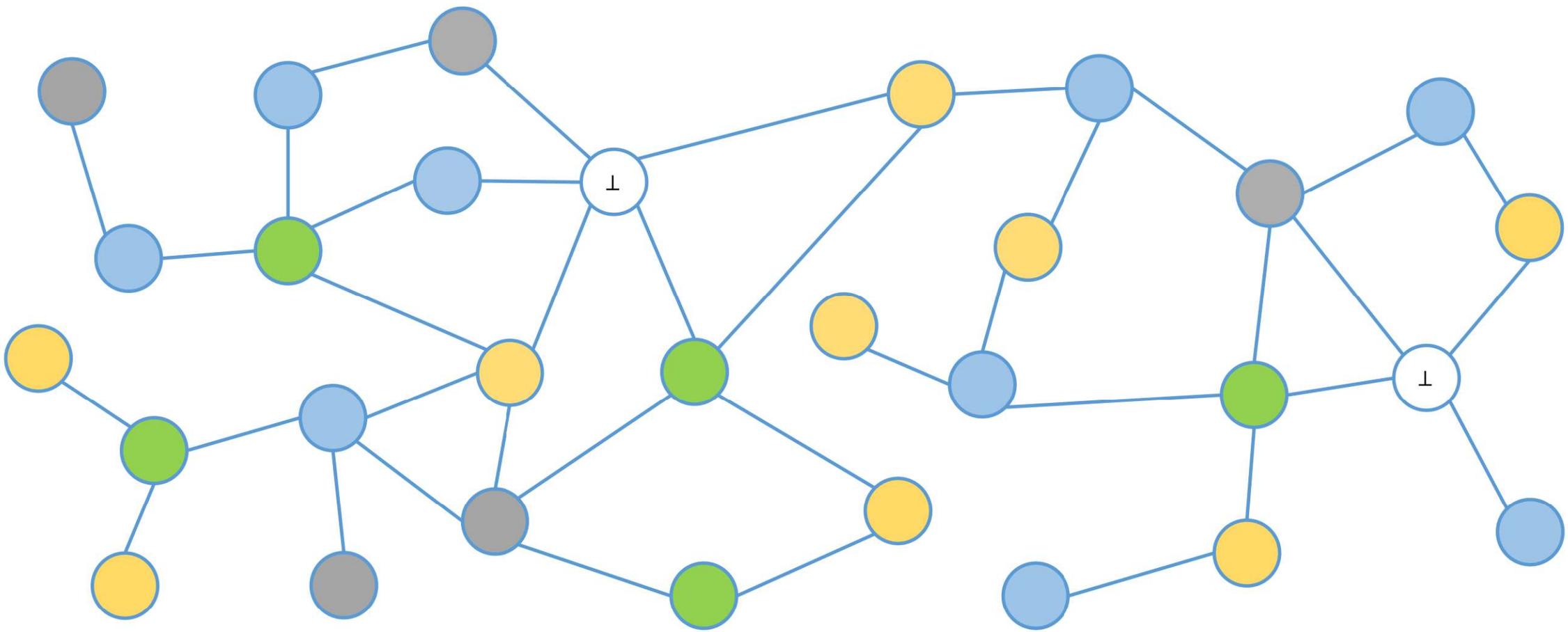
LCLs – locally checkable labeling problems



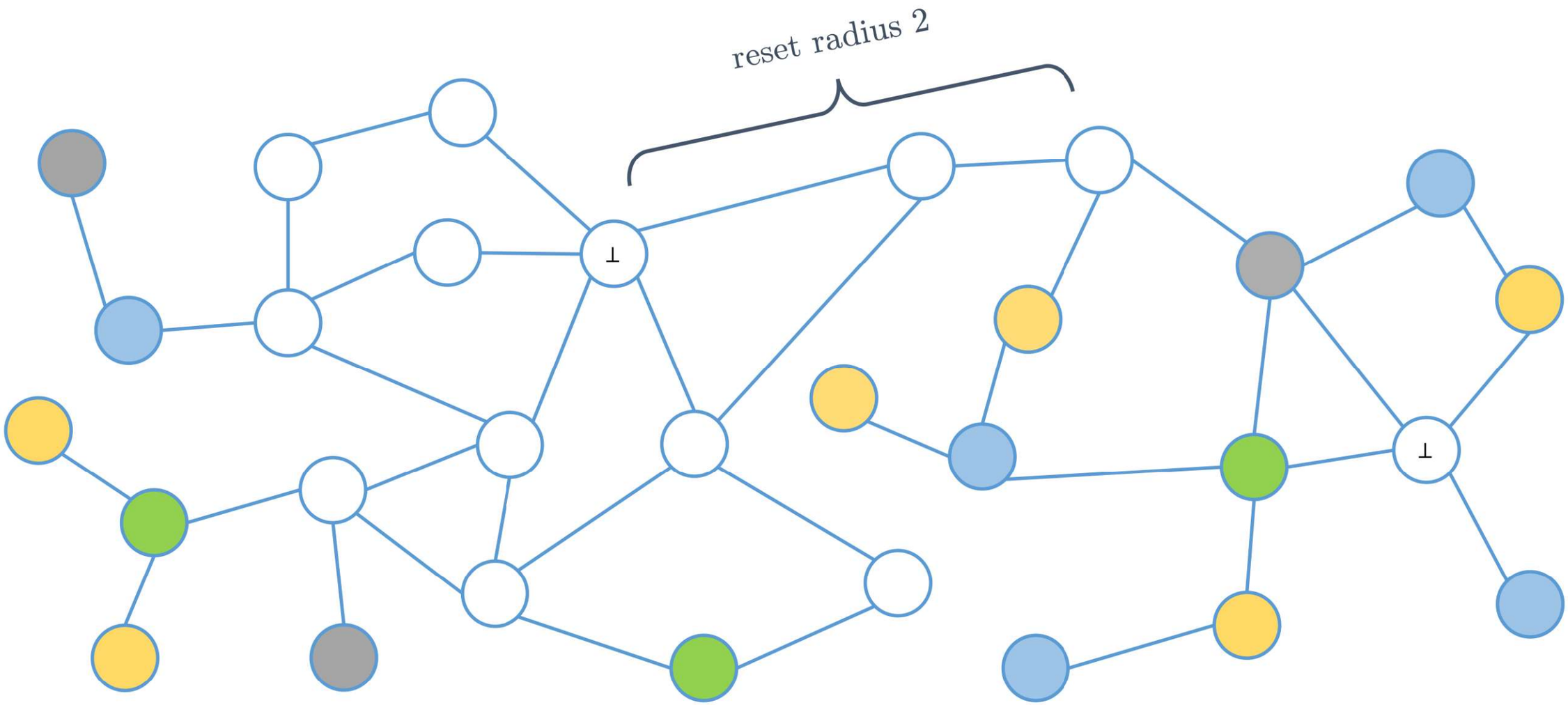
LCLs – locally checkable labeling problems



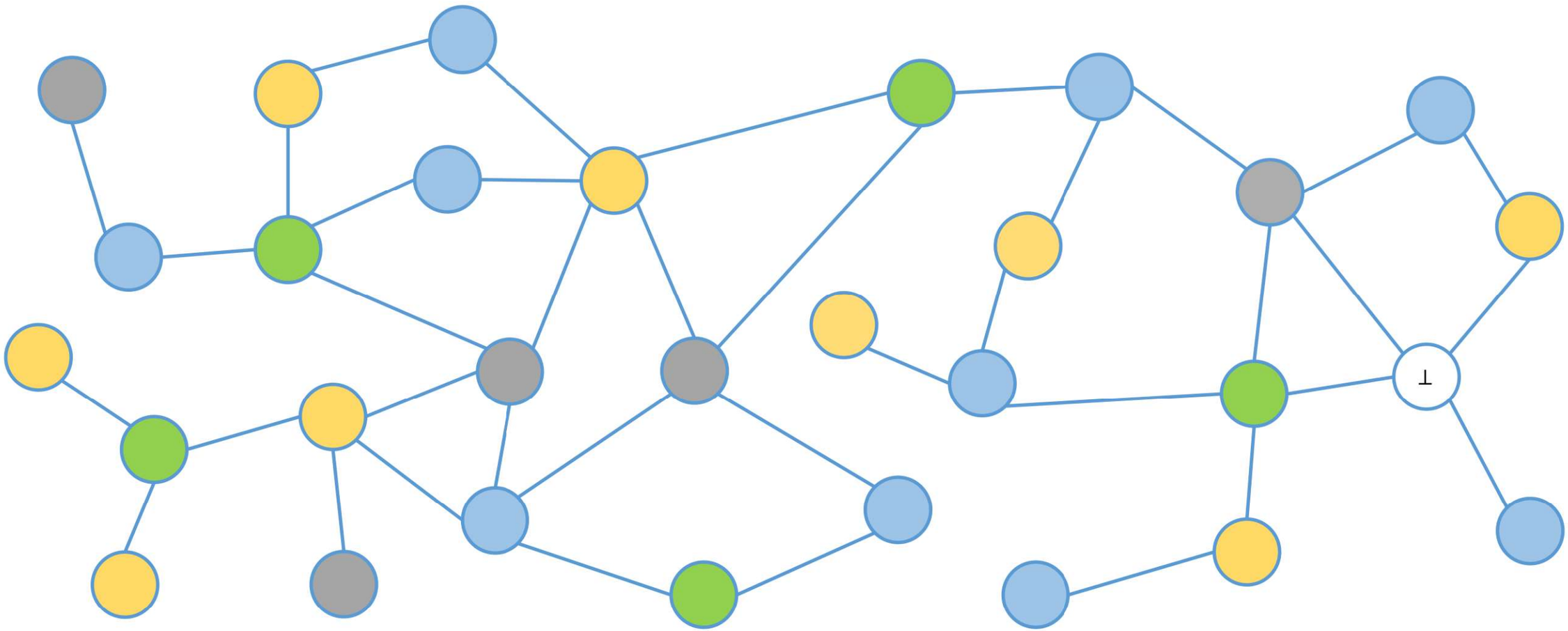
Mending with radius k=2



Mending with radius k=2



Mending with radius k=2

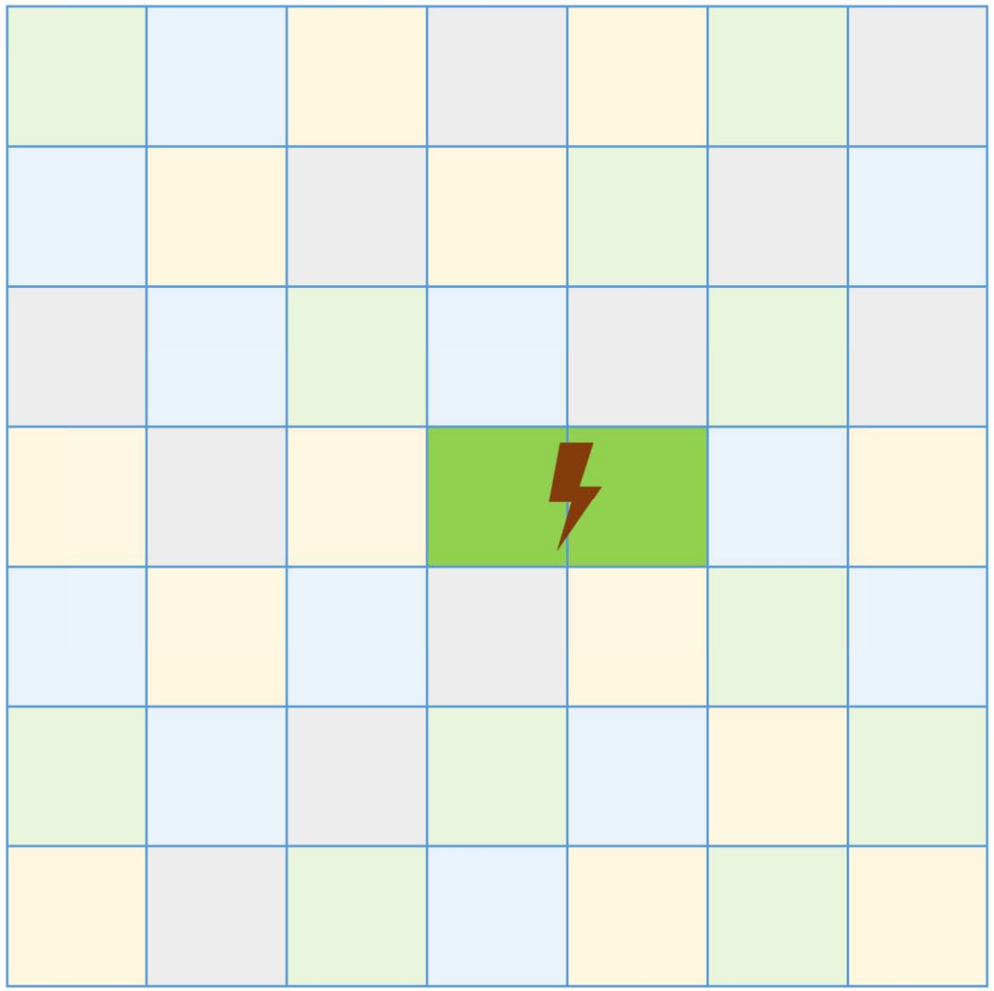


Why mending?

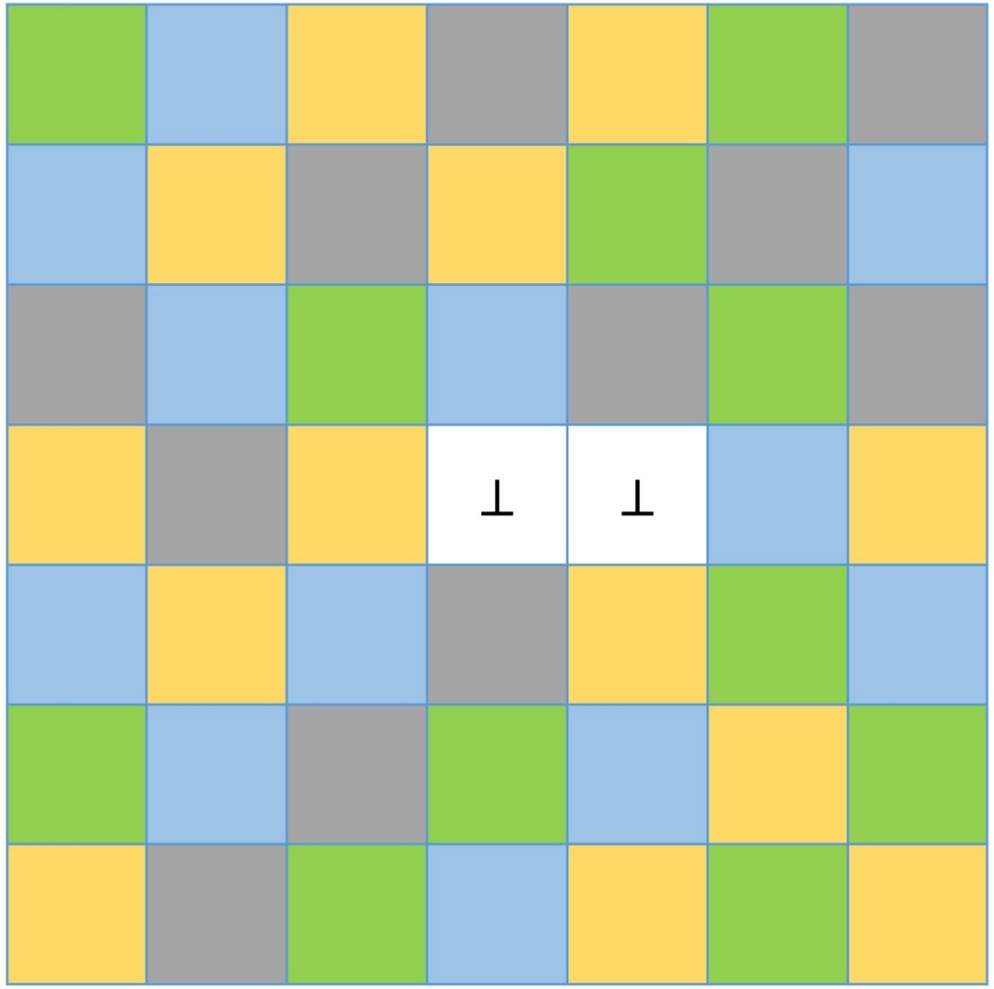
Why mending?

- Mending is a way to respond to failures

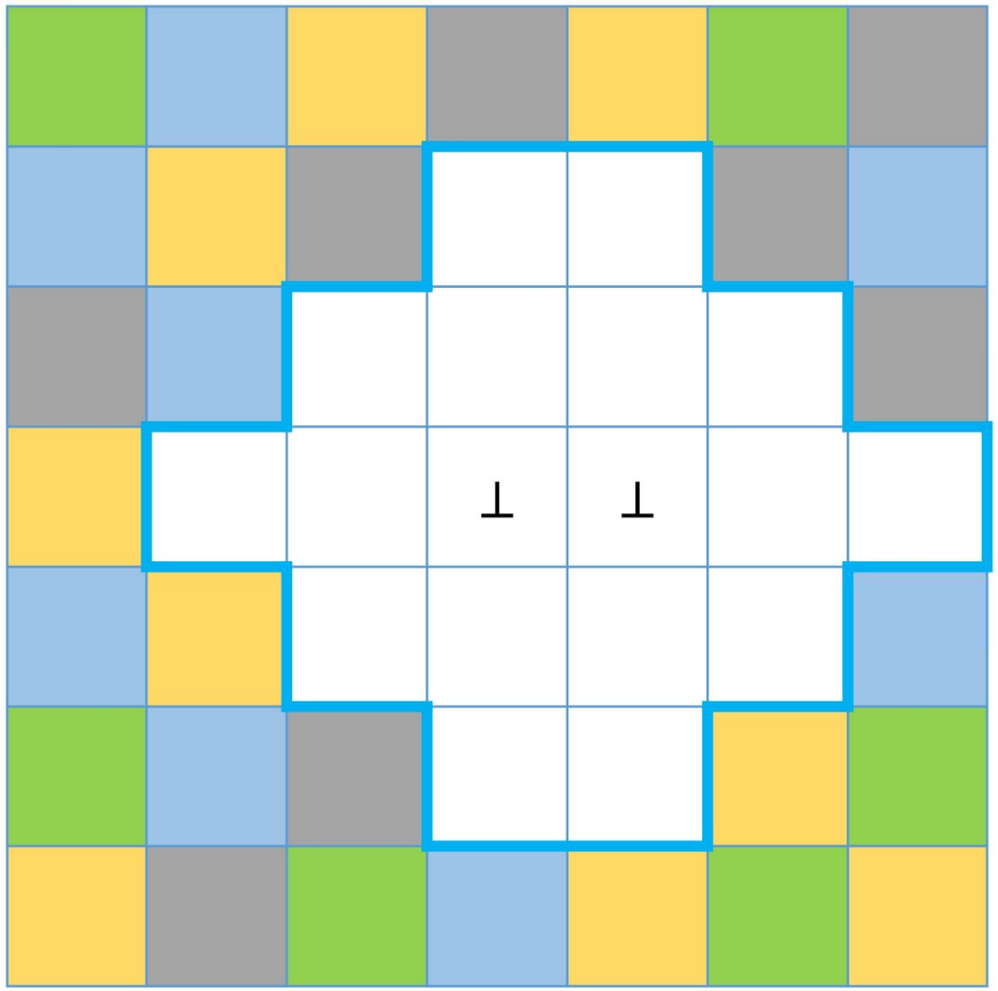
Responding to failures



Responding to failures



Responding to failures



mend local
neighborhood

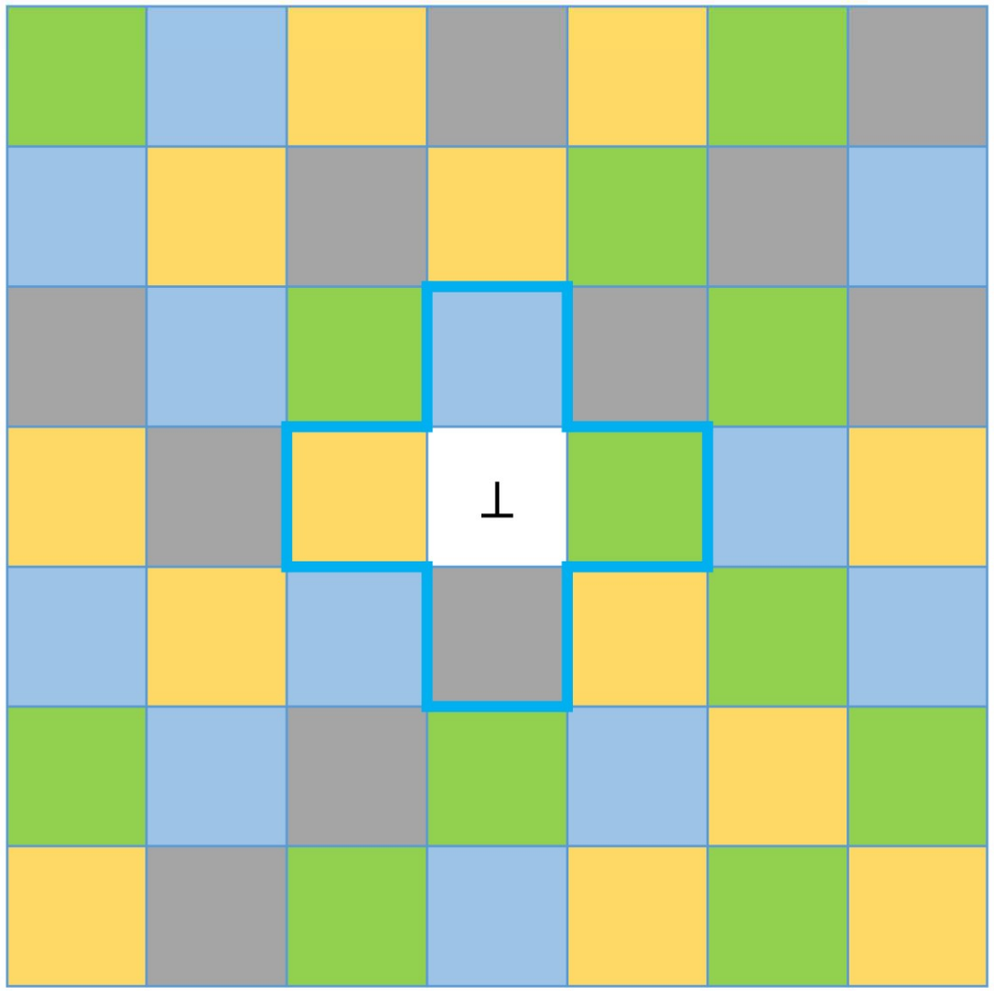
Why mending?

- Mending is a way to respond to failures
- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL

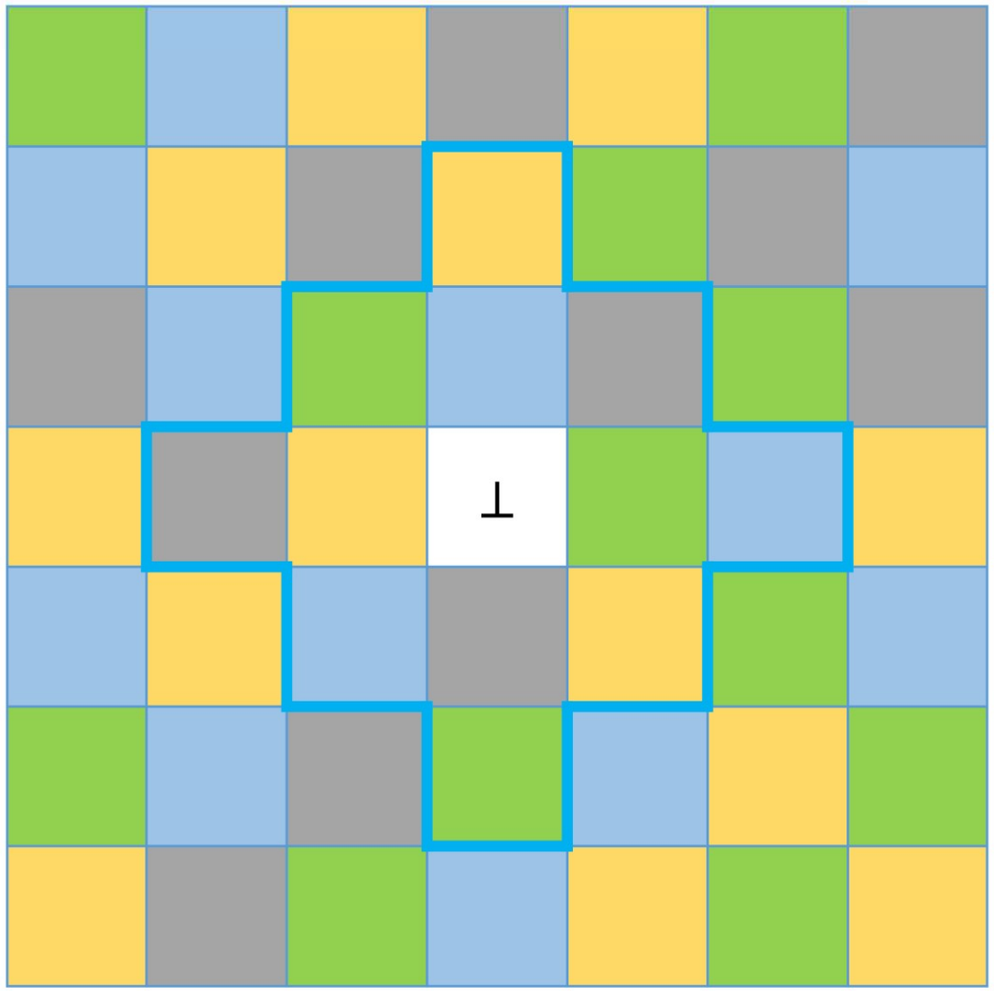
Why mending?

- Mending is a way to respond to failures
- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL
- Computer-assisted algorithm design

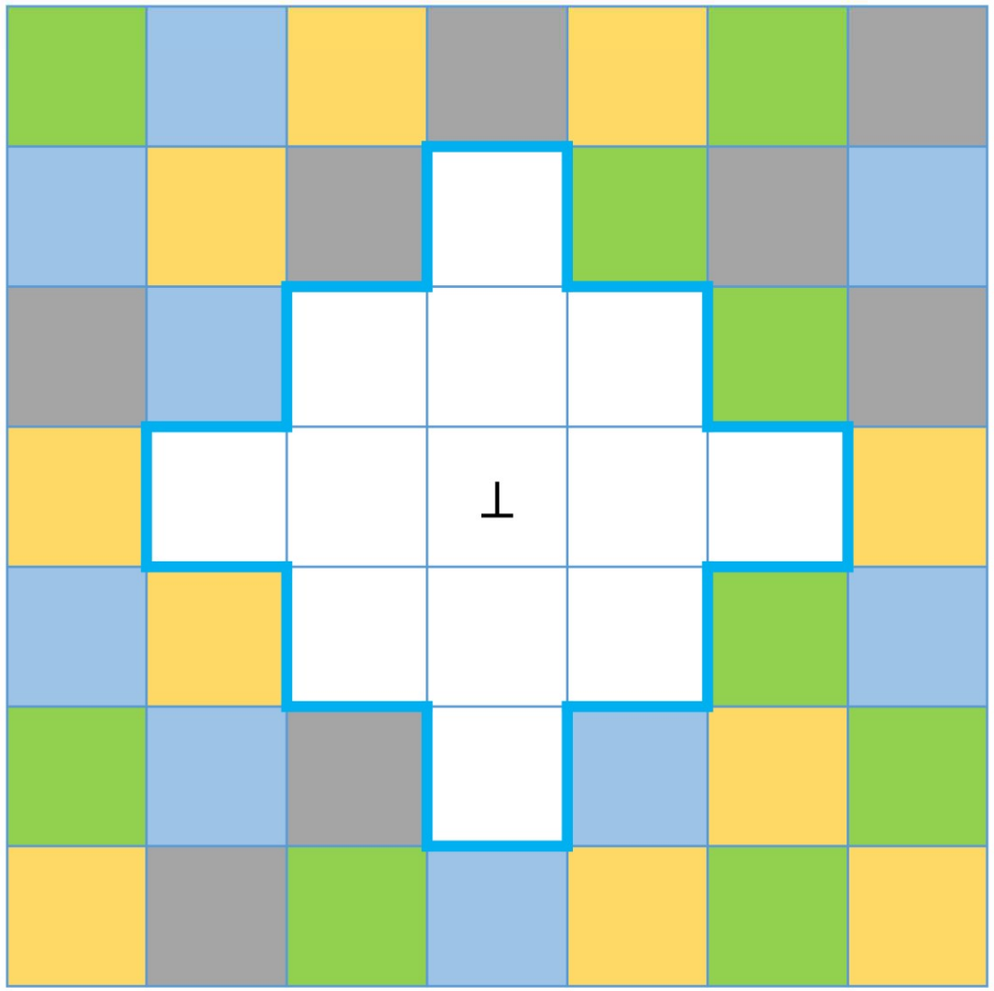
Computer-assisted algorithm design



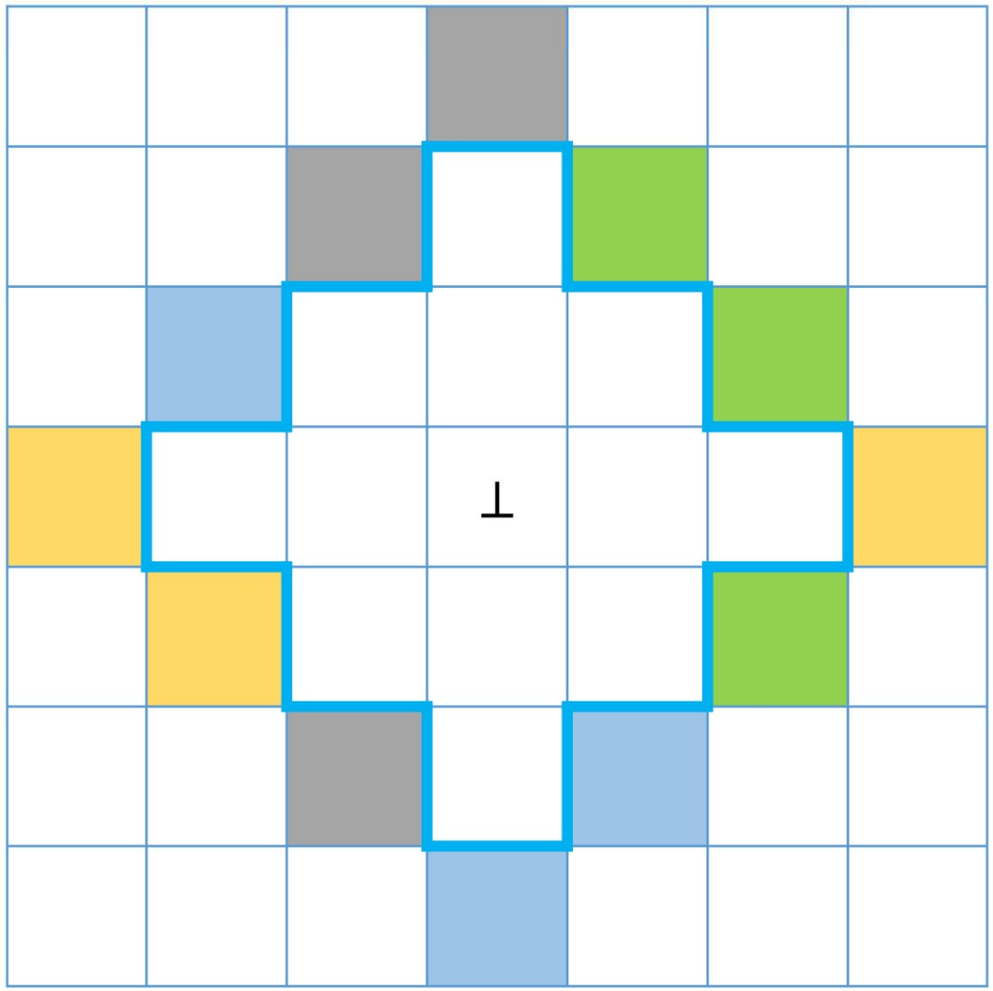
Computer-assisted algorithm design



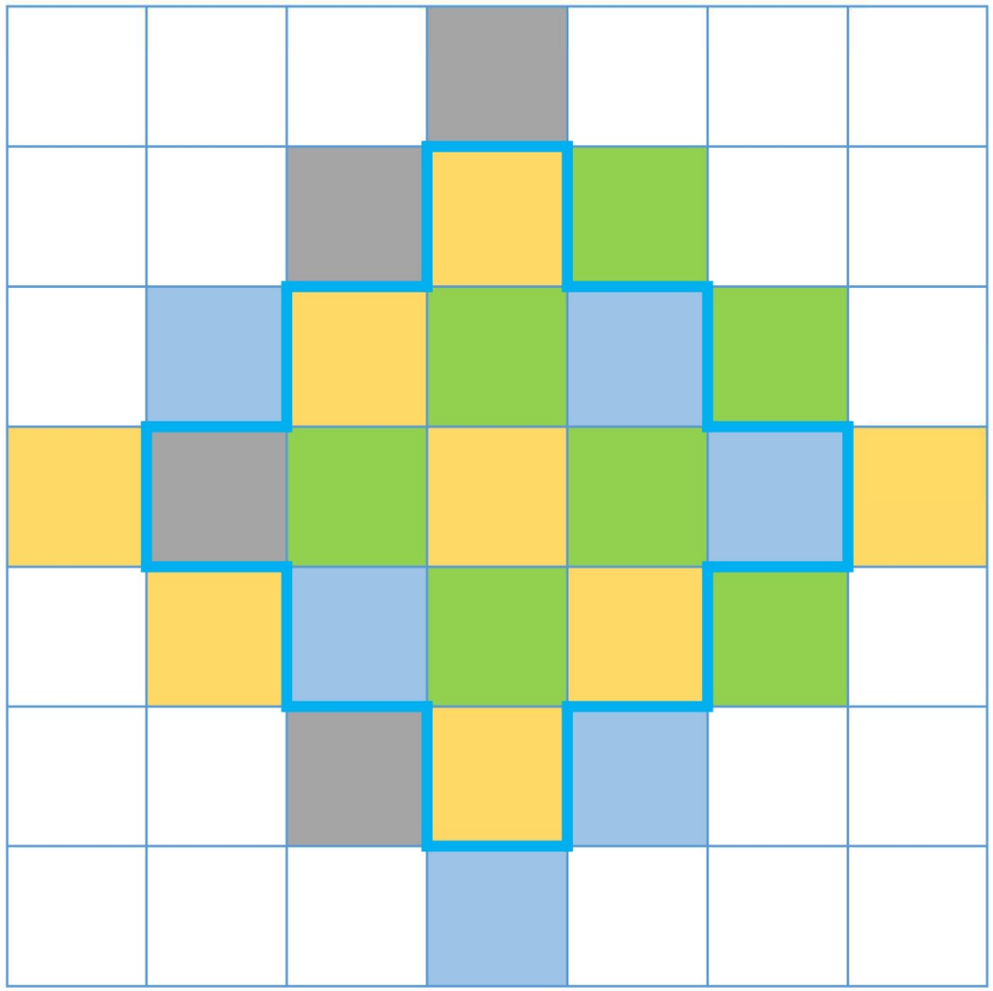
Computer-assisted algorithm design



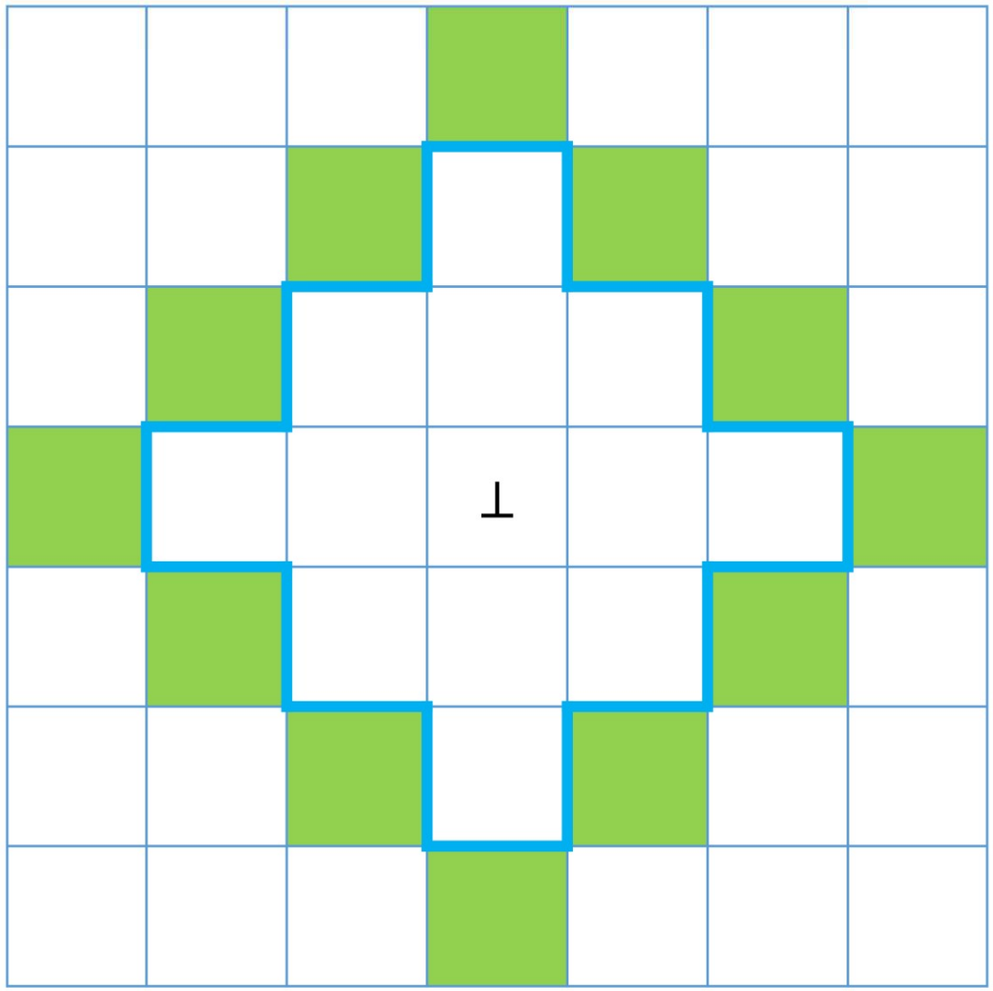
Computer-assisted algorithm design



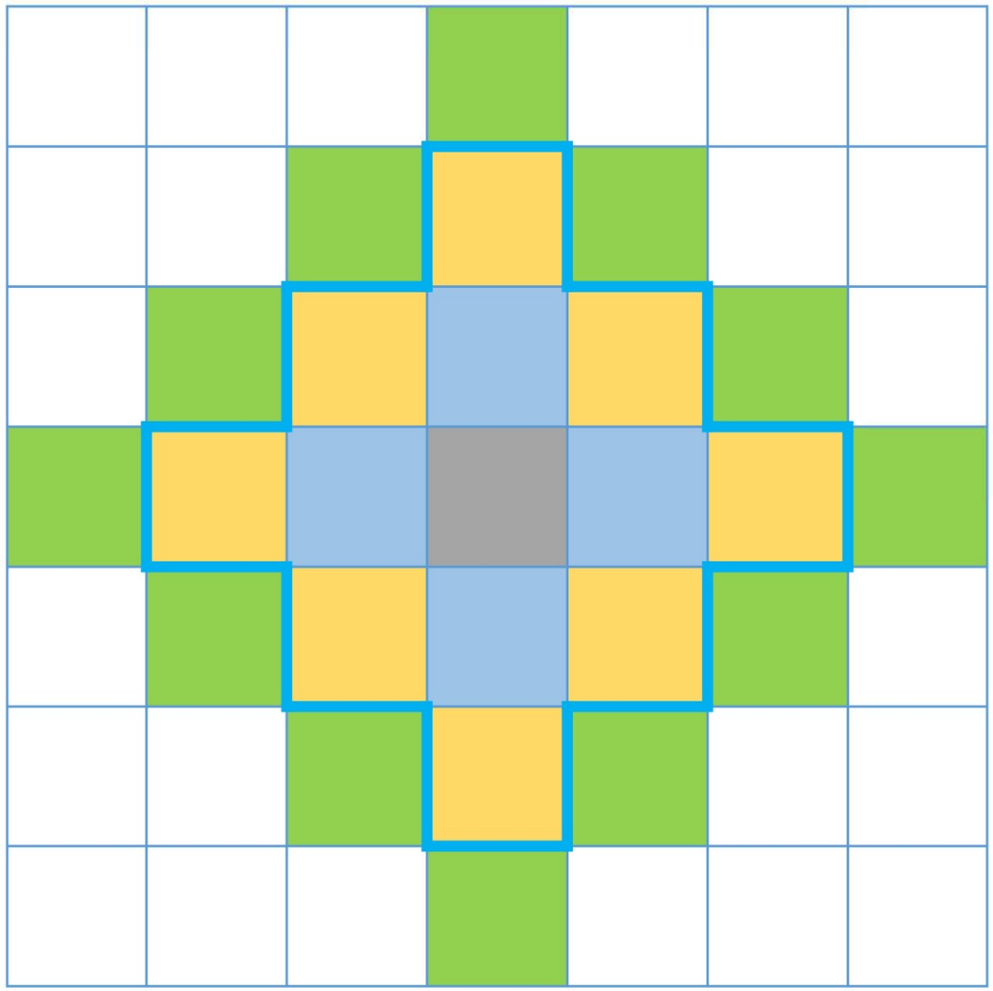
Computer-assisted algorithm design



Computer-assisted algorithm design



Computer-assisted algorithm design



From mendability to solvability

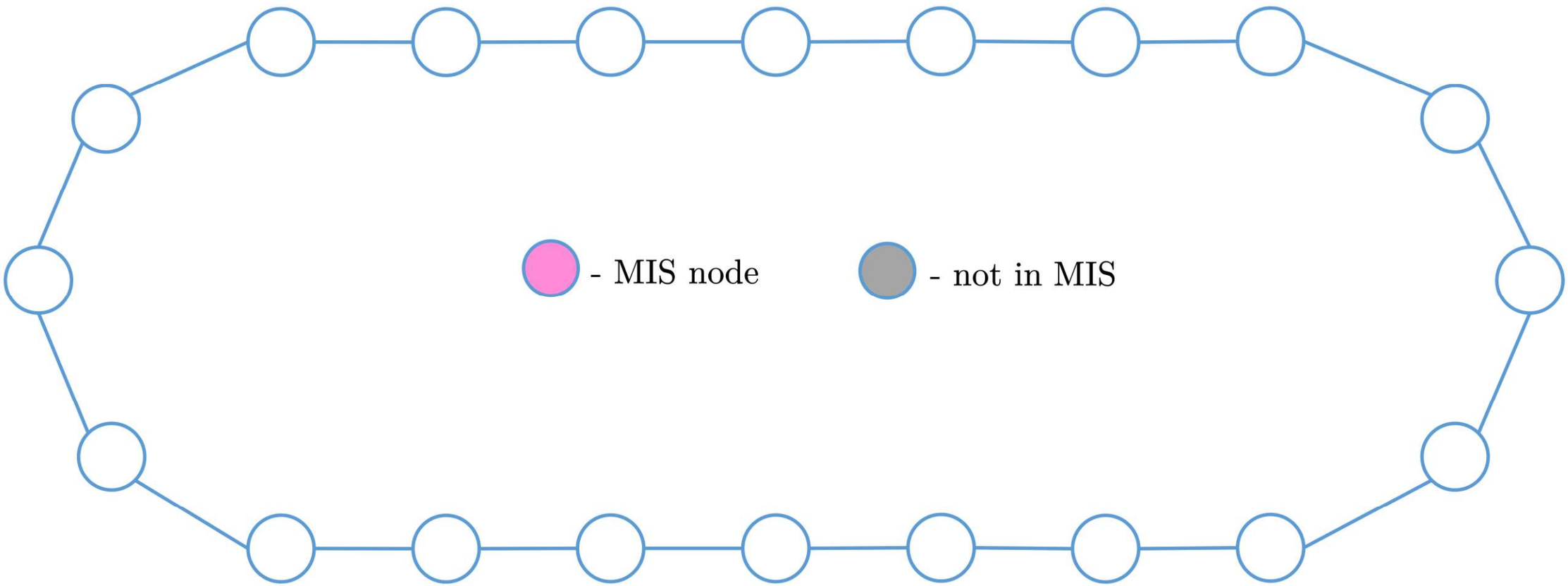
Constant-radius mendable
$\Longrightarrow O(\log^* n)$-solvable in LOCAL

From mendability to solvability

- Solving the Maximal Independent Set problem (MIS)
- Mending radius $k = 1$
- Checkability radius $\mathrm{r} = 2$

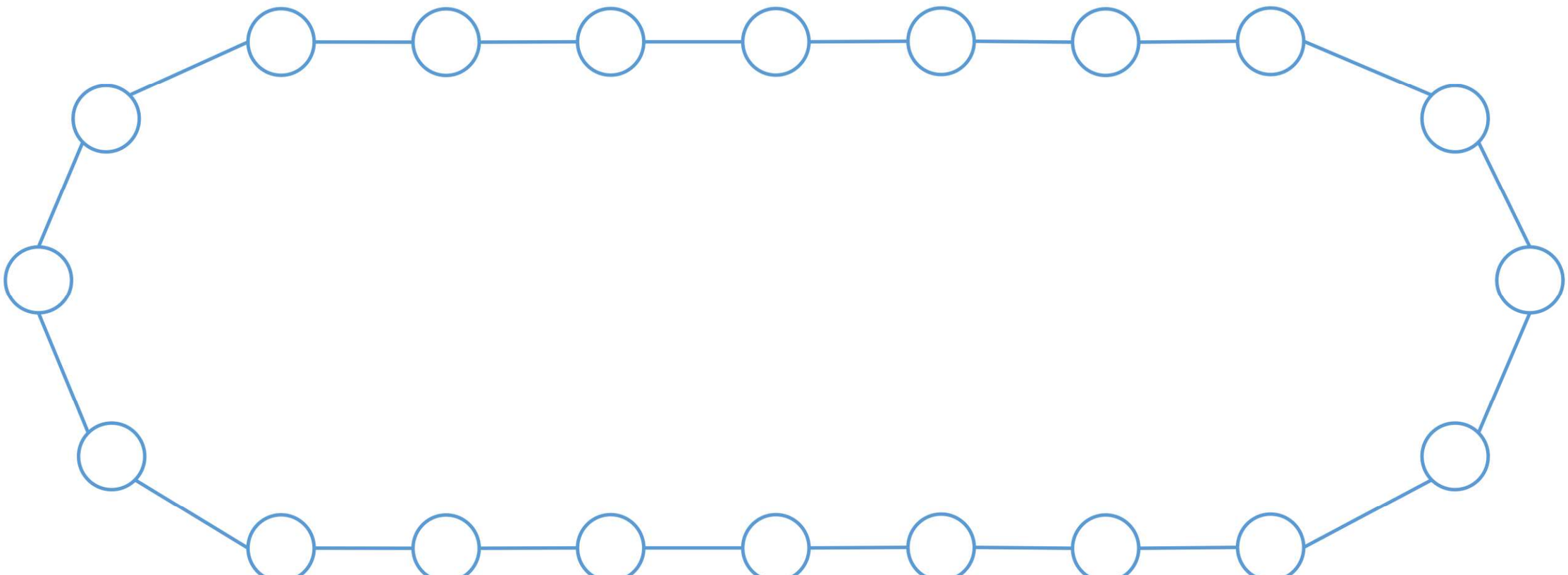
From mendability to solvability

- Maximal Independent Set (MIS) problem



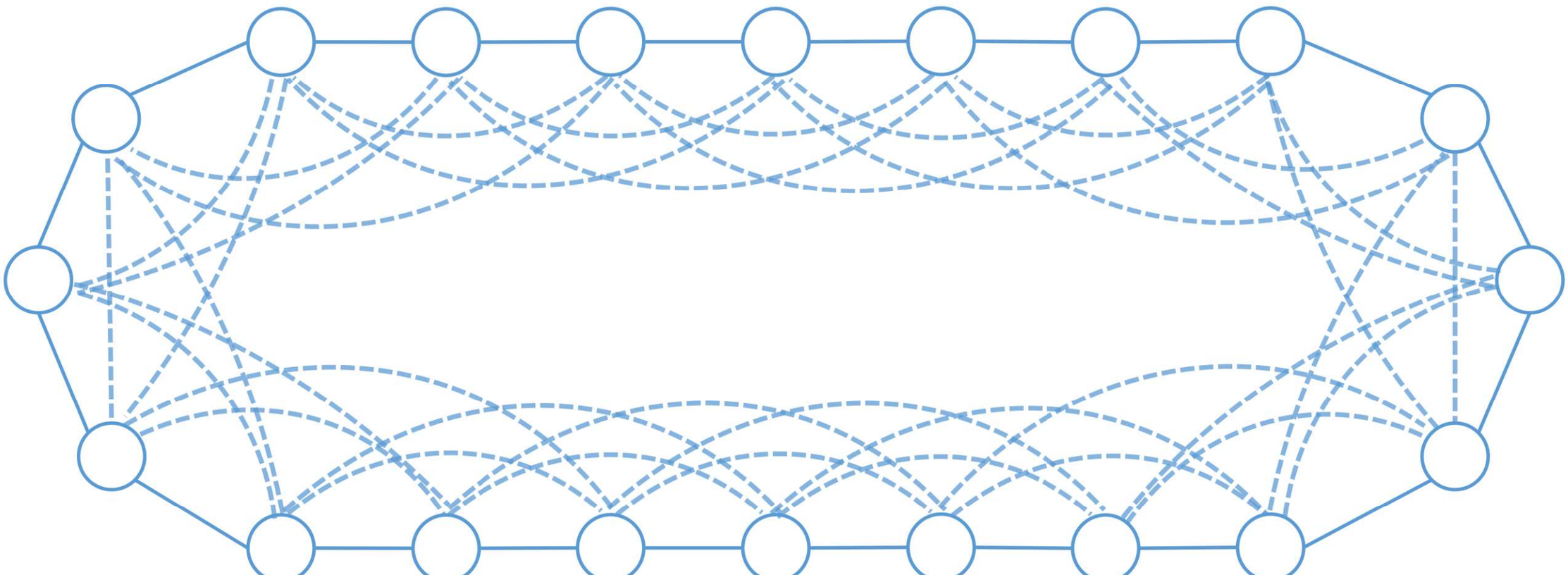
From mendability to solvability

- Compute distance-$(2k+1)$ coloring with Δ^{2k+r} colors



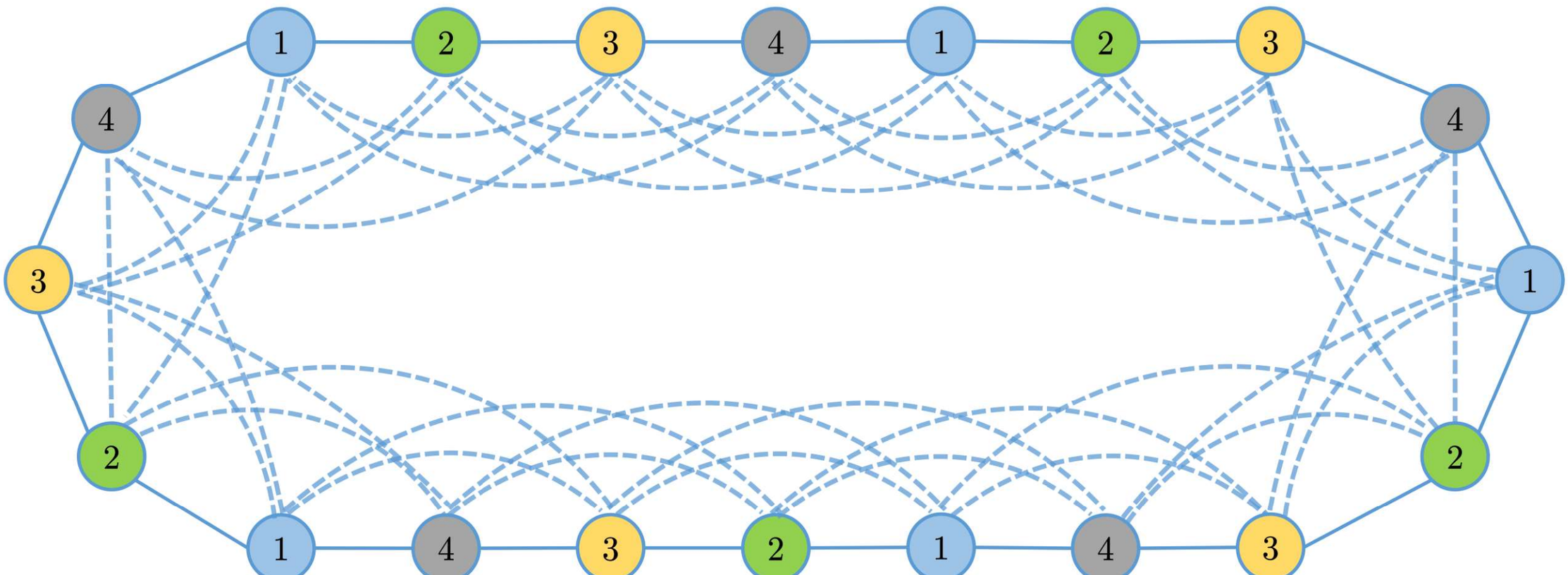
From mendability to solvability

- Distance-3 coloring



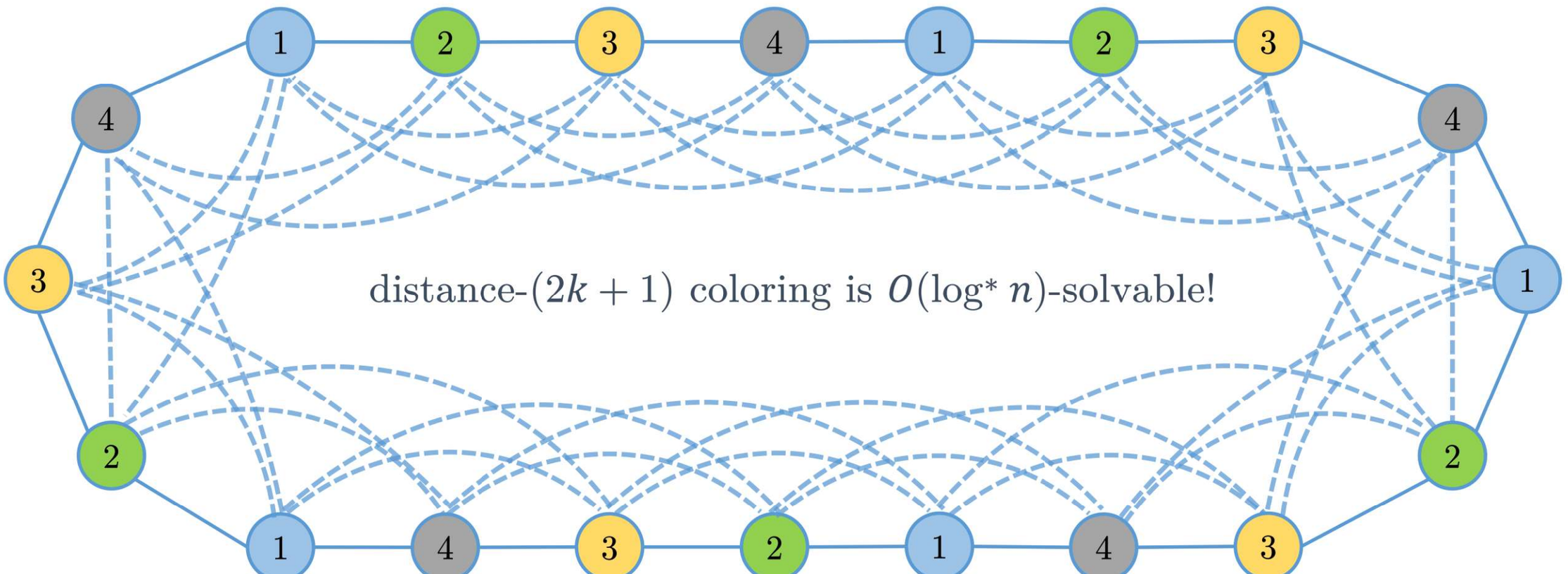
From mendability to solvability

- Distance-3 coloring



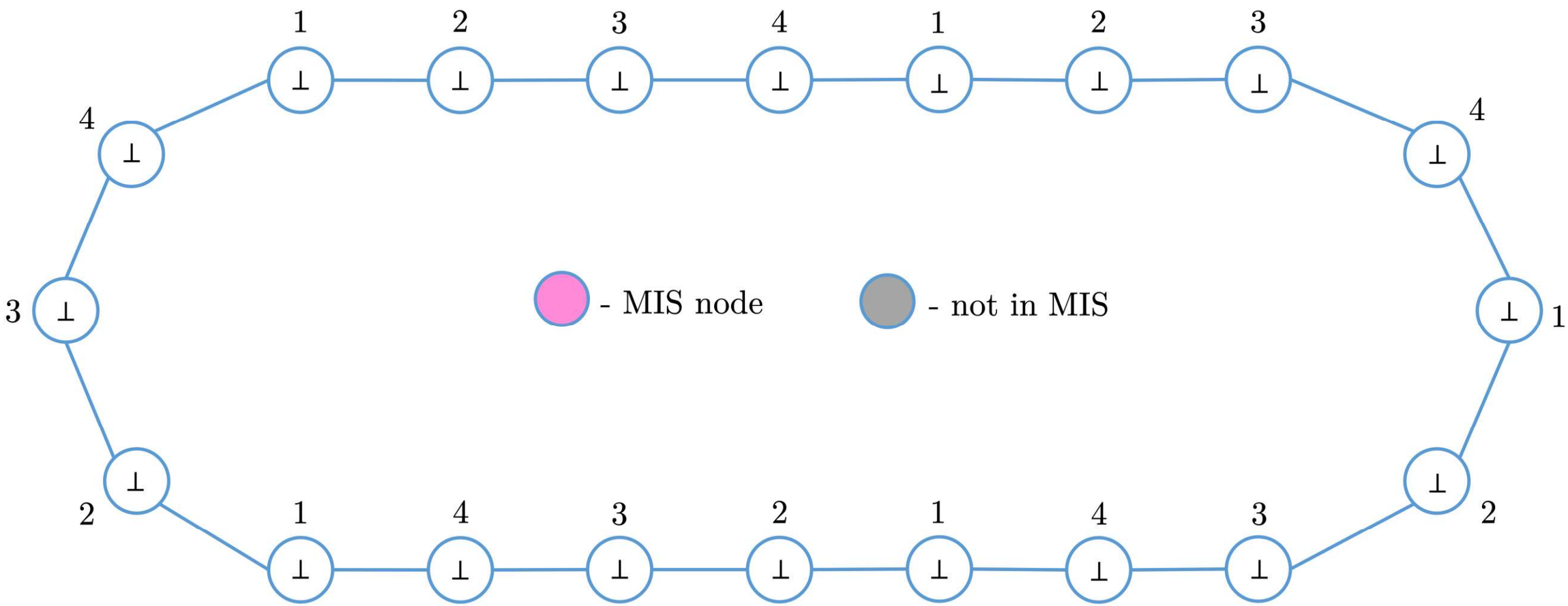
From mendability to solvability

- Distance-3 coloring



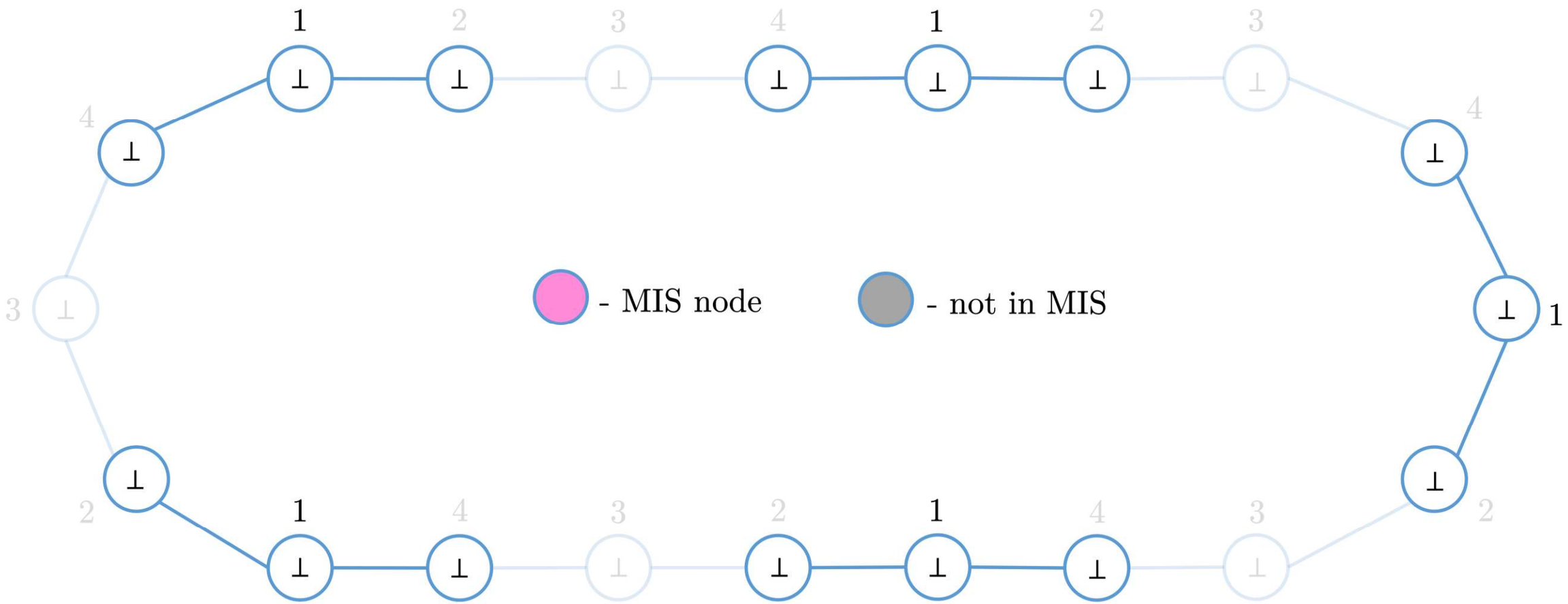
From mendability to solvability

- Mend by color classes

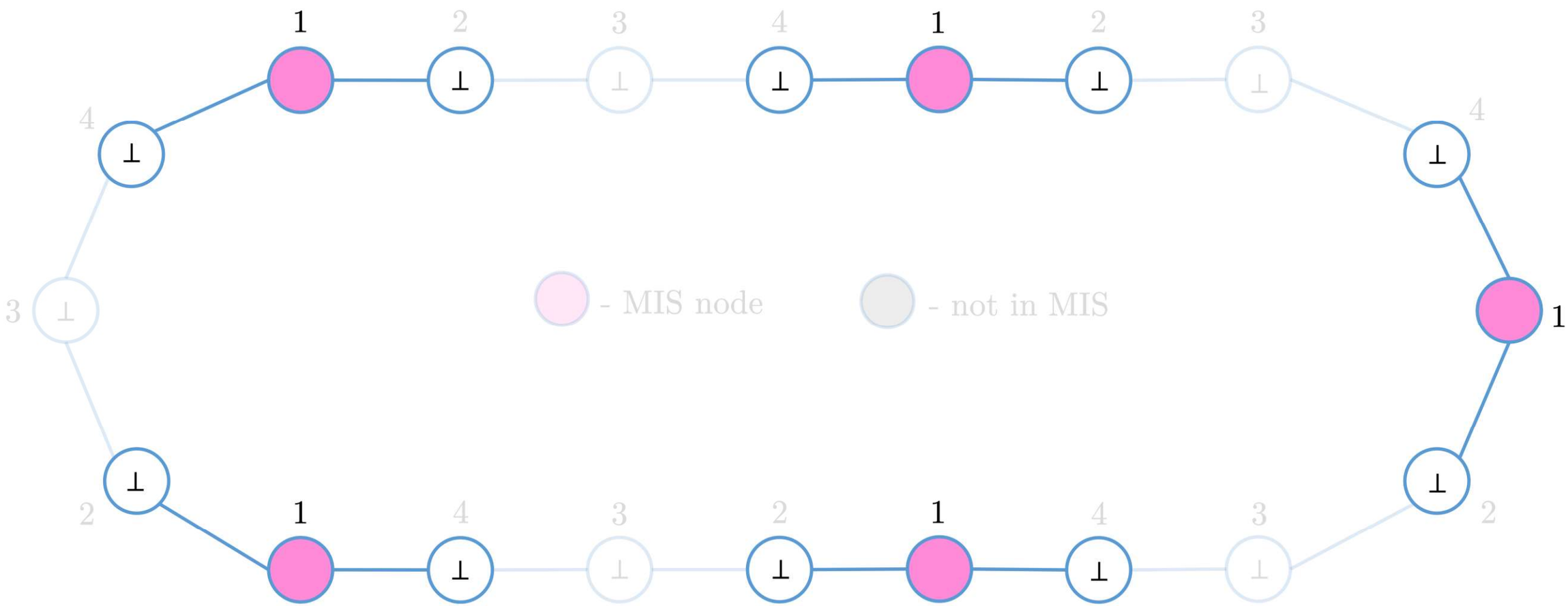


From mendability to solvability

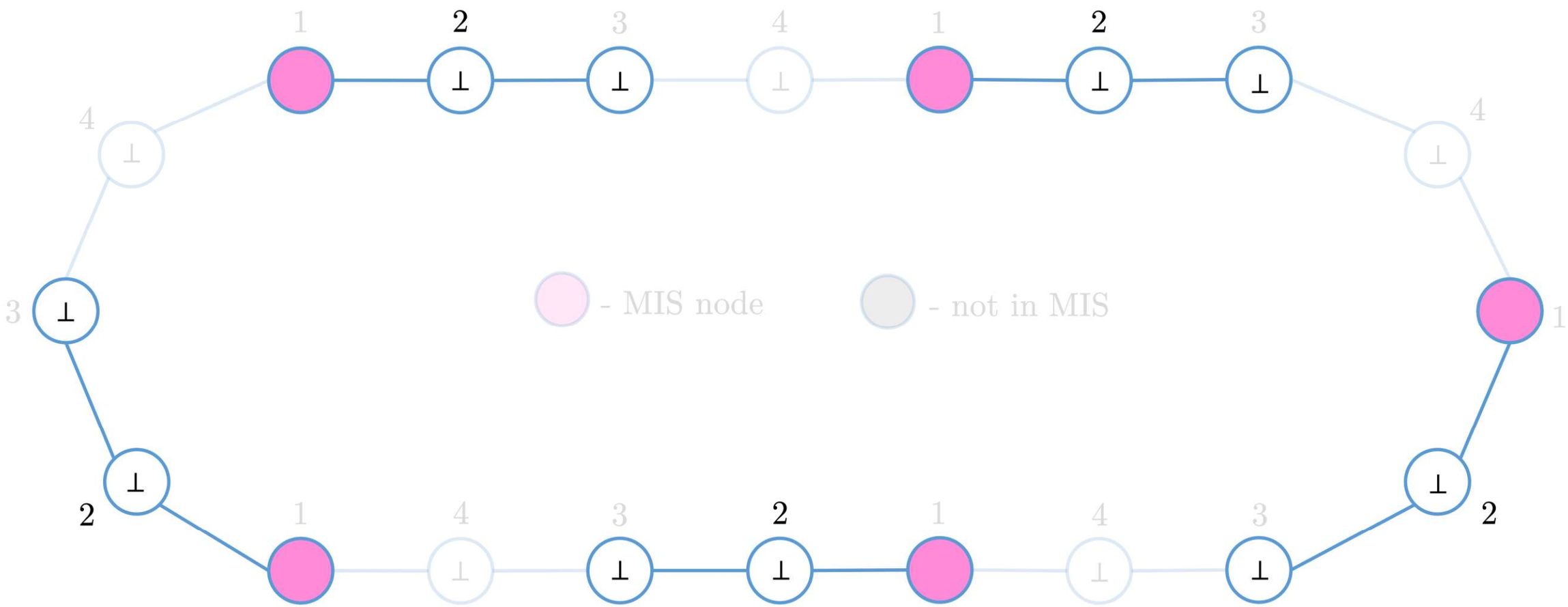
- Solving MIS with mending radius 1



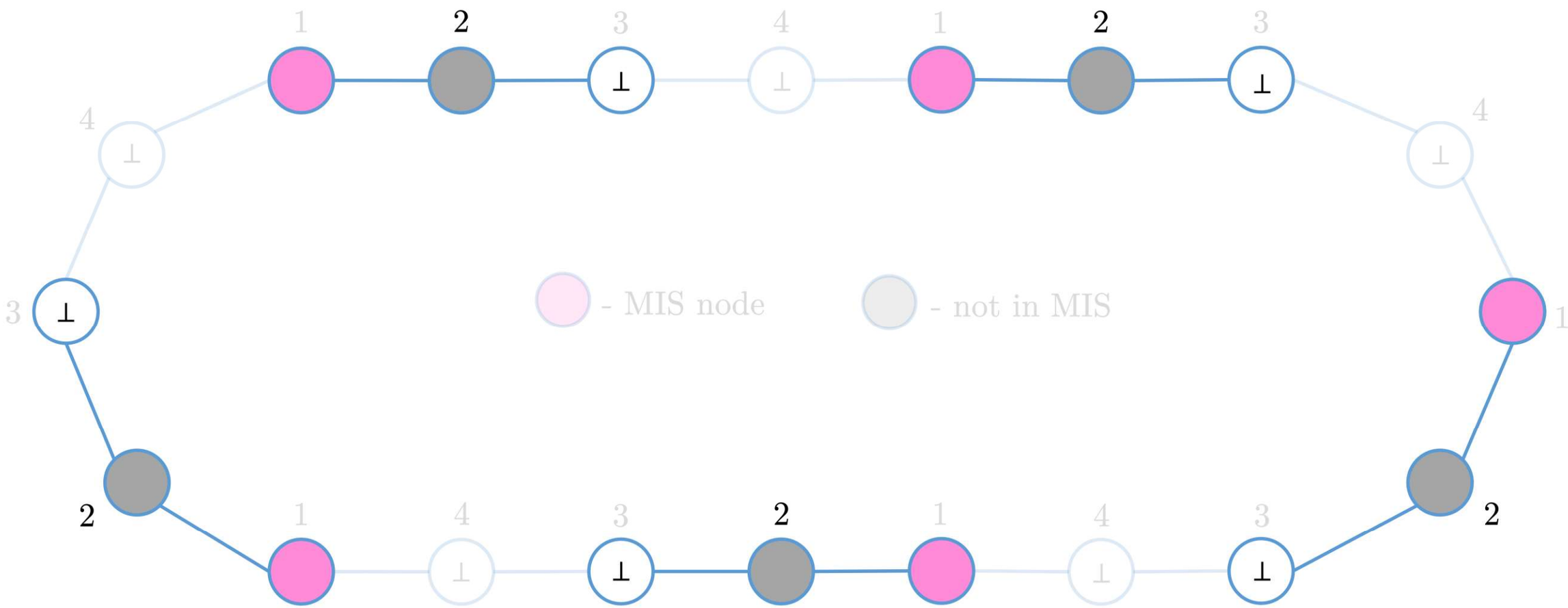
From mendability to solvability



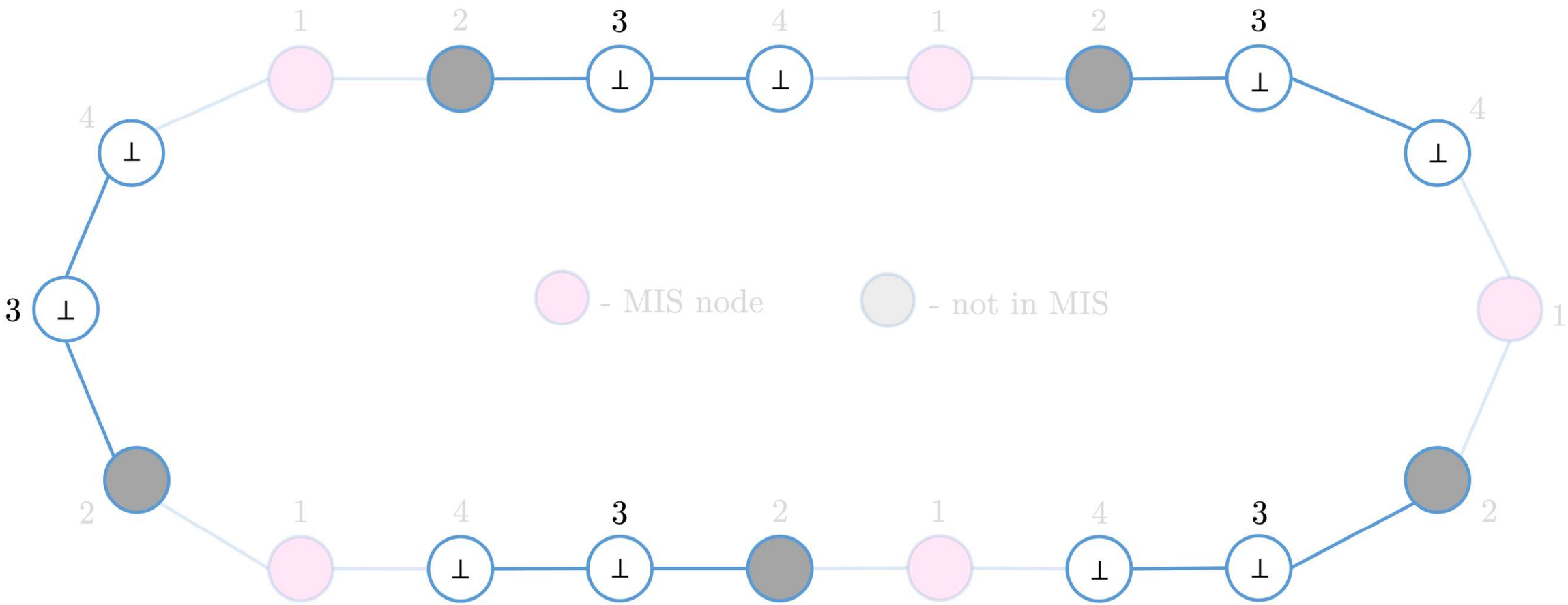
From mendability to solvability



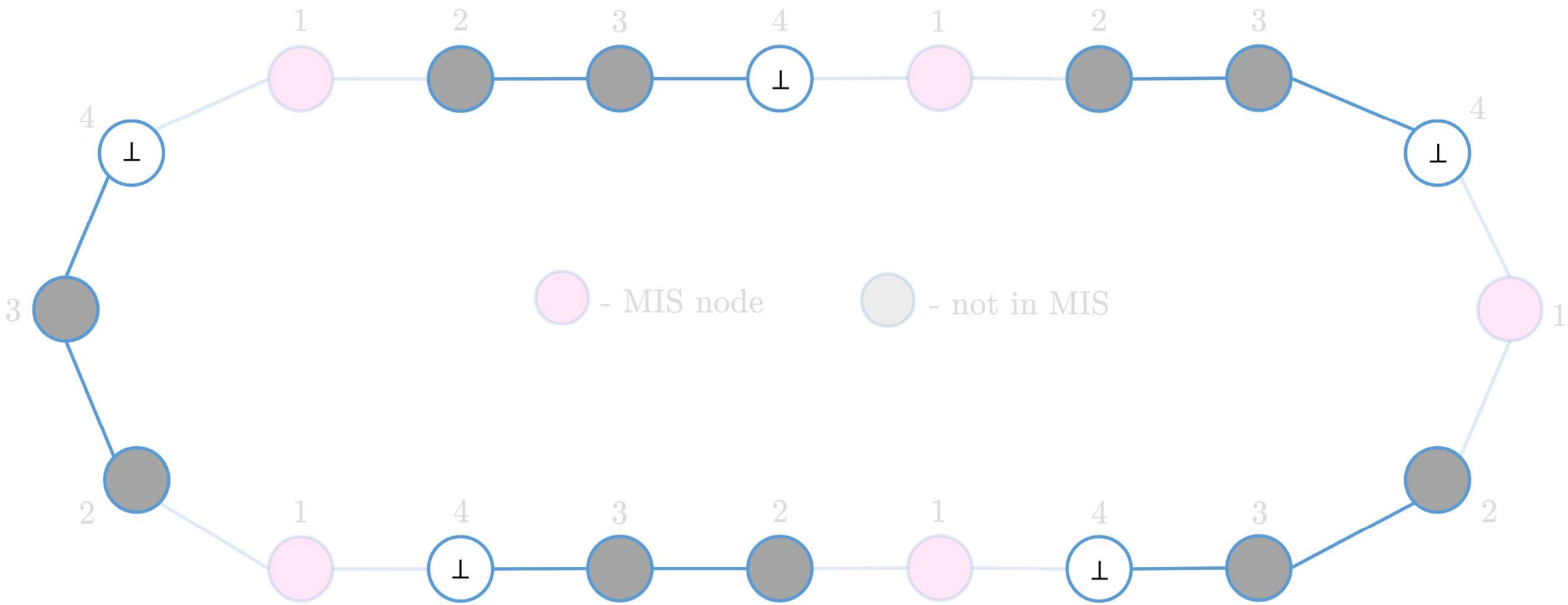
From mendability to solvability



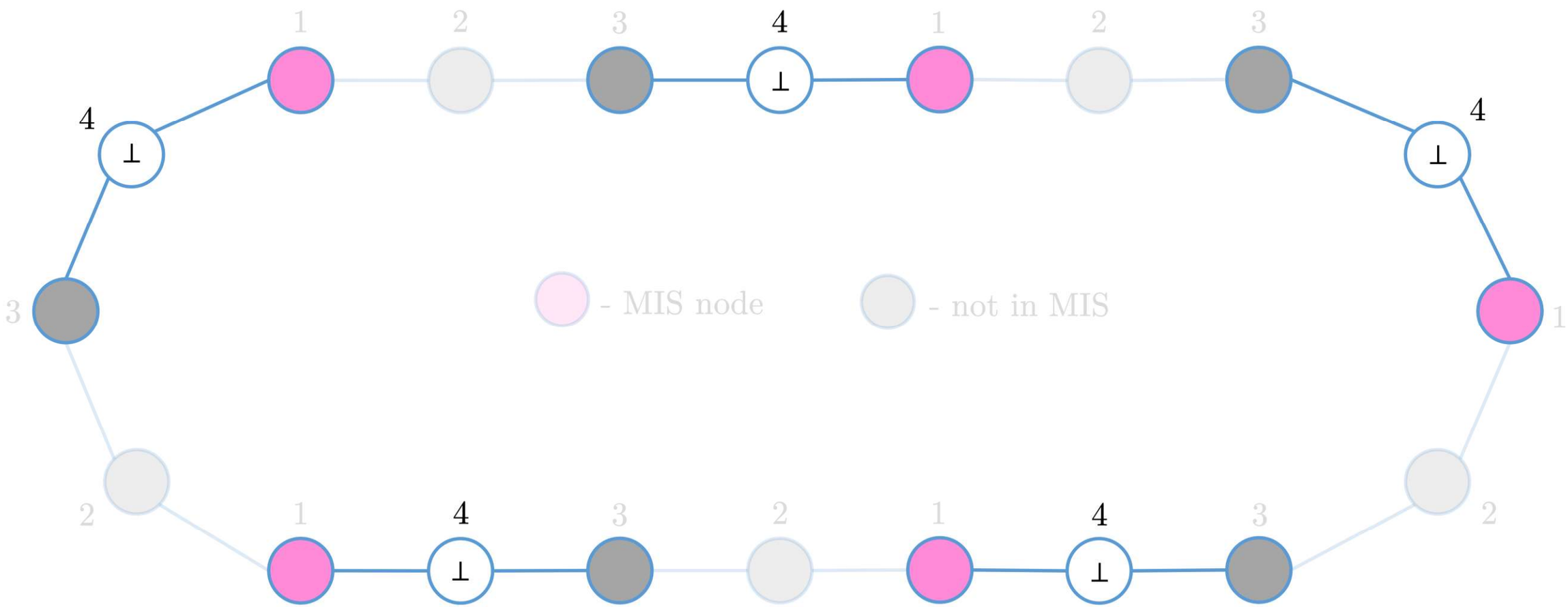
From mendability to solvability



From mendability to solvability

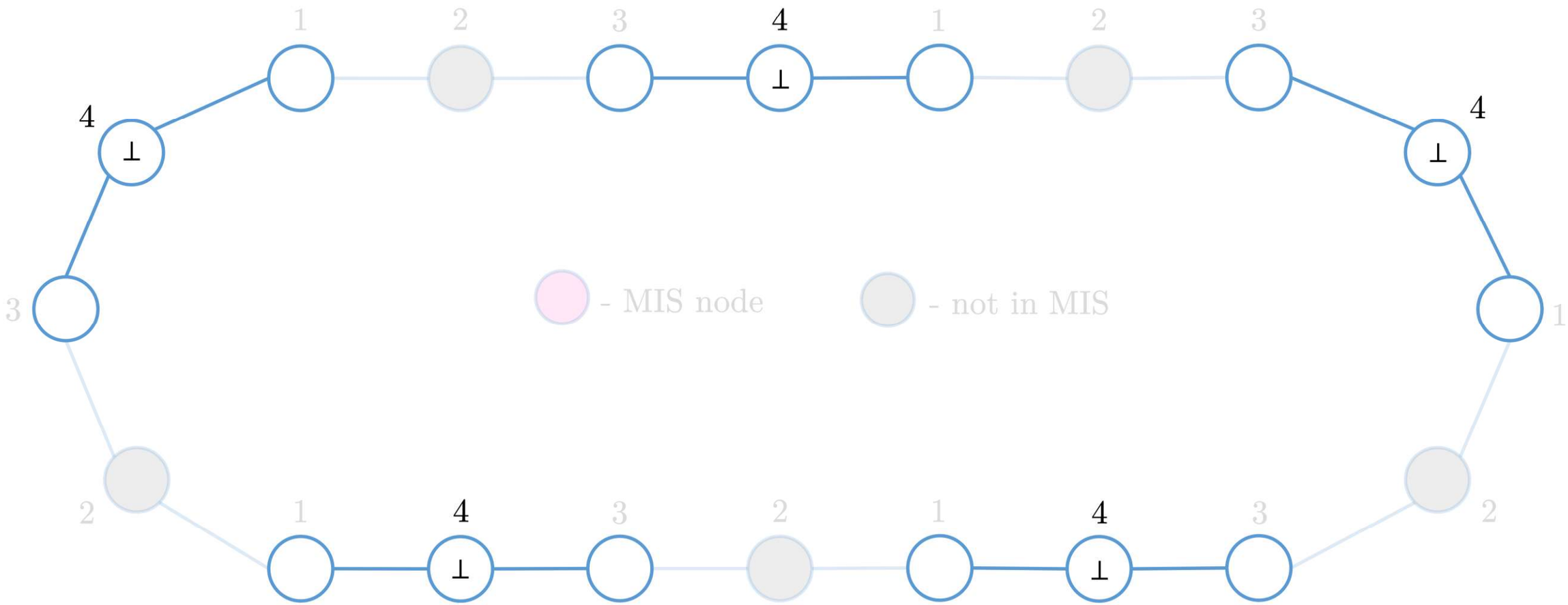


From mendability to solvability



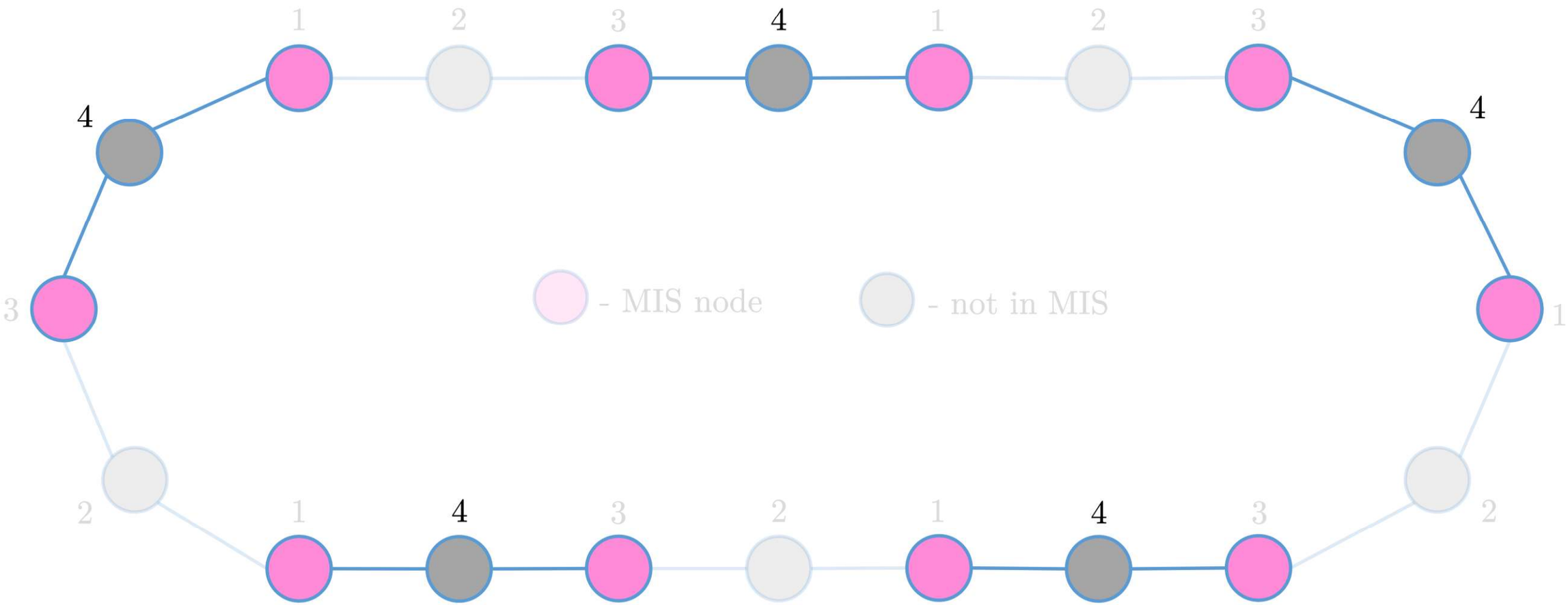
From mendability to solvability

- Undo labeling in the radius 1 neighborhood

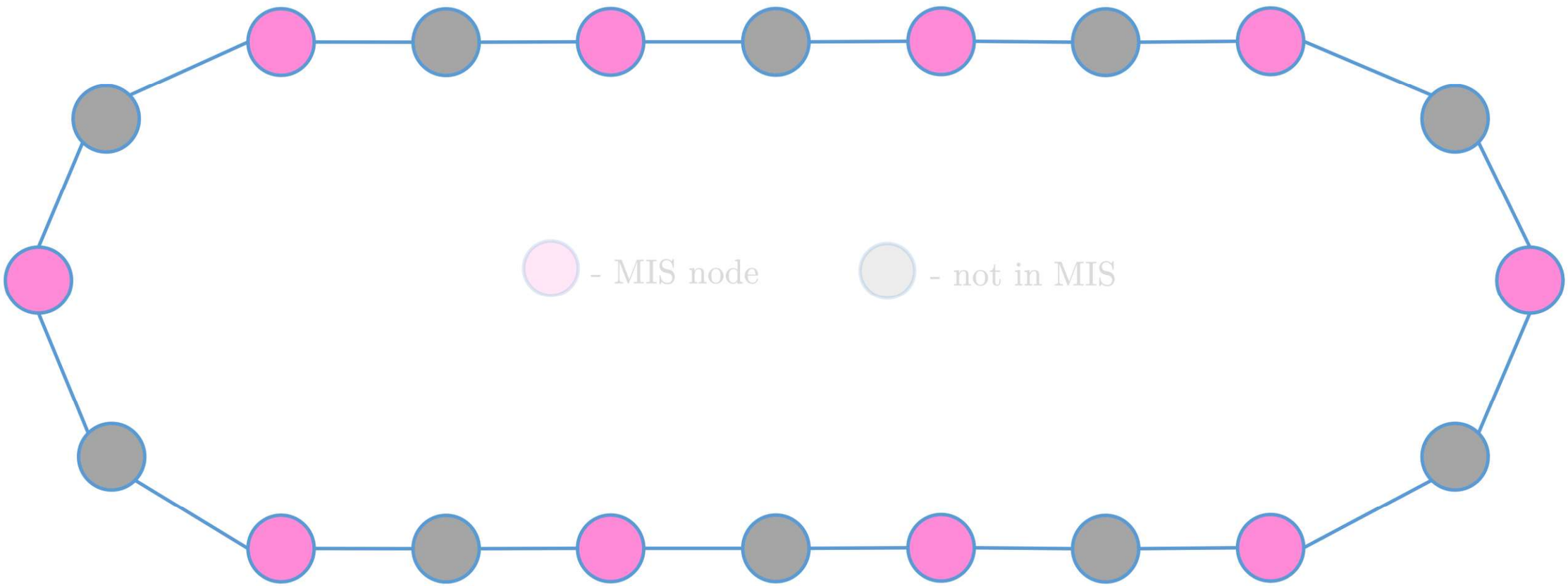


From mendability to solvability

- Mend



From mendability to solvability



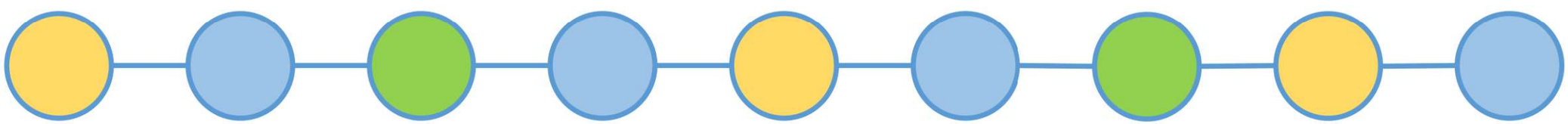
Does solvability imply mendability?

Does solvability imply mendability?

2-color a path with labels A and B

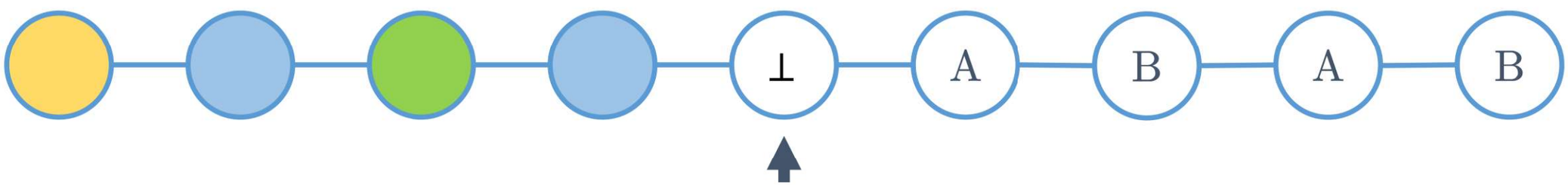
OR

3-color a path with colors green, blue and yellow



Does solvability imply mendability?

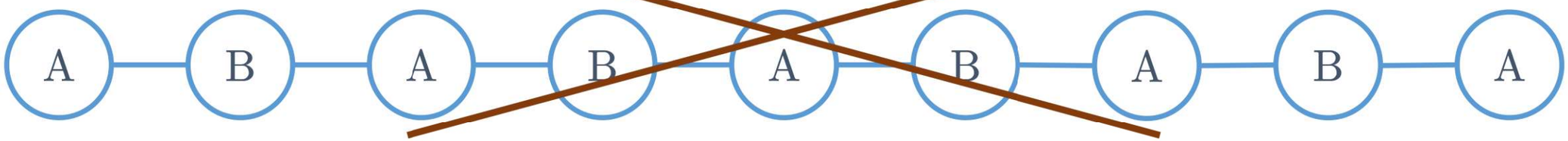
- Solvable in $O(\log^* n)$ in LOCAL
- Mending needs radius $\Omega(n)$



Does solvability imply mendability?

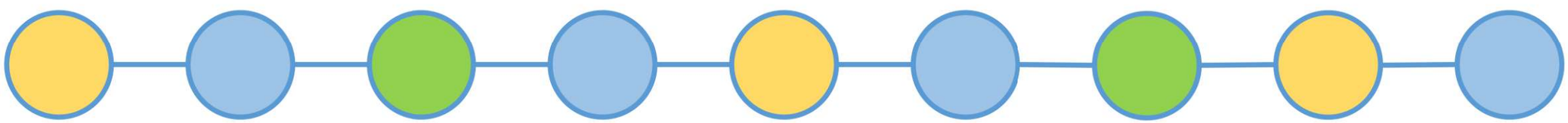
- A restriction of the problem is radius-$O(1)$ mendable!

2-color a path with labels A and B



OR

3-color a path with colors green, blue and yellow



Does solvability imply mendability?

Any $O(\log^* n)$-solvable problem on paths and cycles can be restricted such that

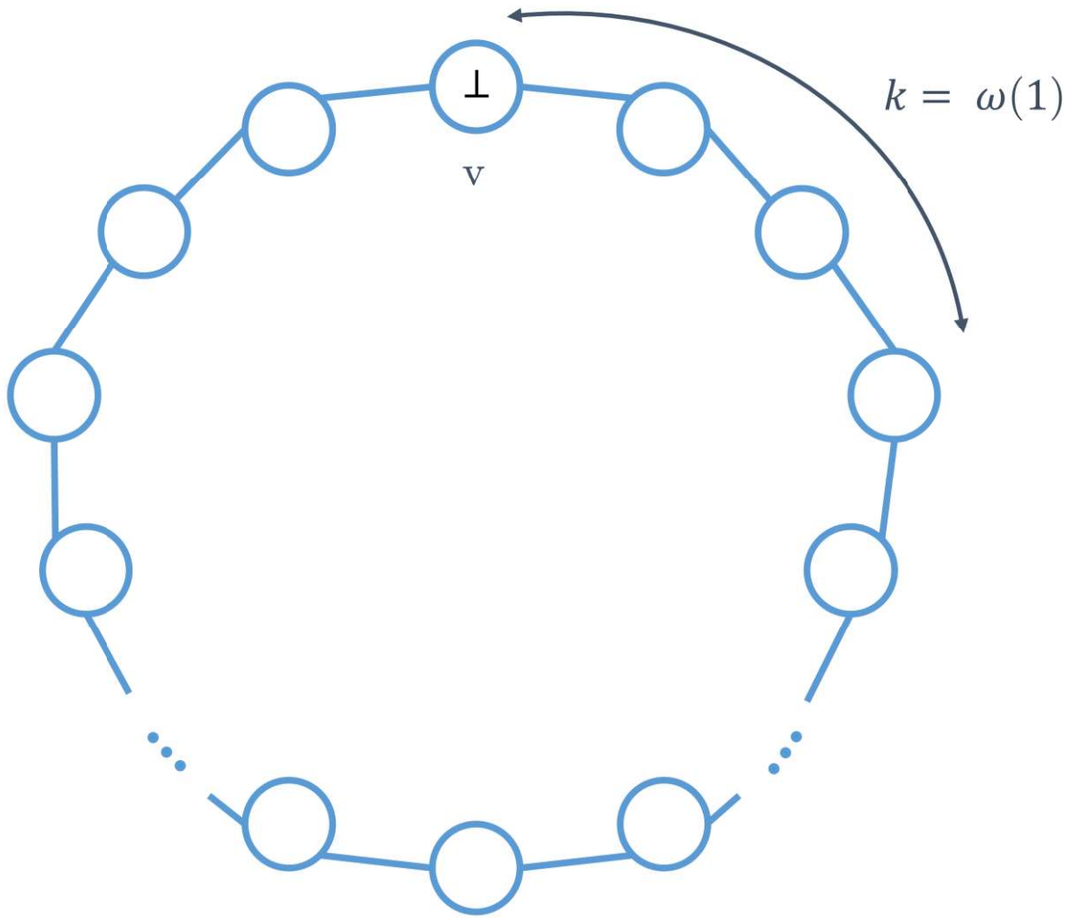
- it is still $O(\log^* n)$-solvable and
- it is radius-$O(1)$ mendable.

Complexity landscape of mending

On cycles, problems either
have mending radius
$O(1)$ or $\Omega(n)$

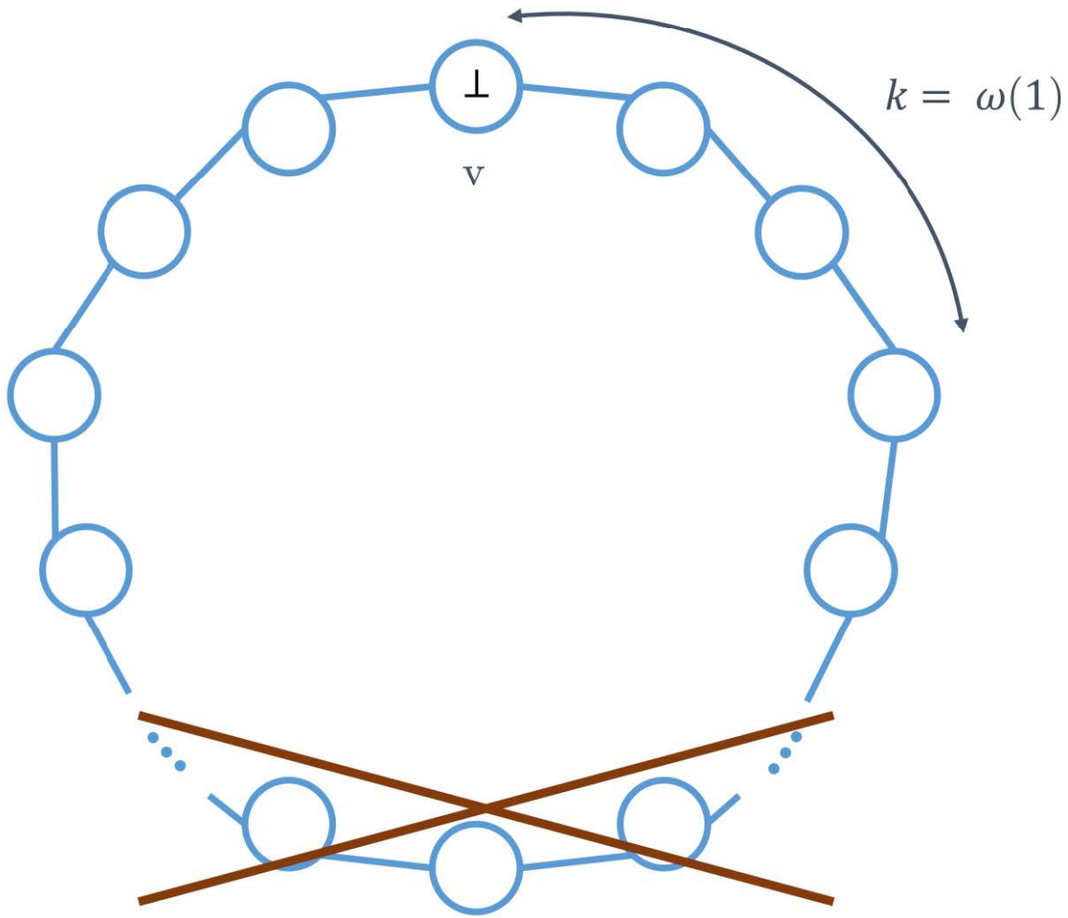
Mending in cycles

- Given: problem with mending radius $\omega(1)$



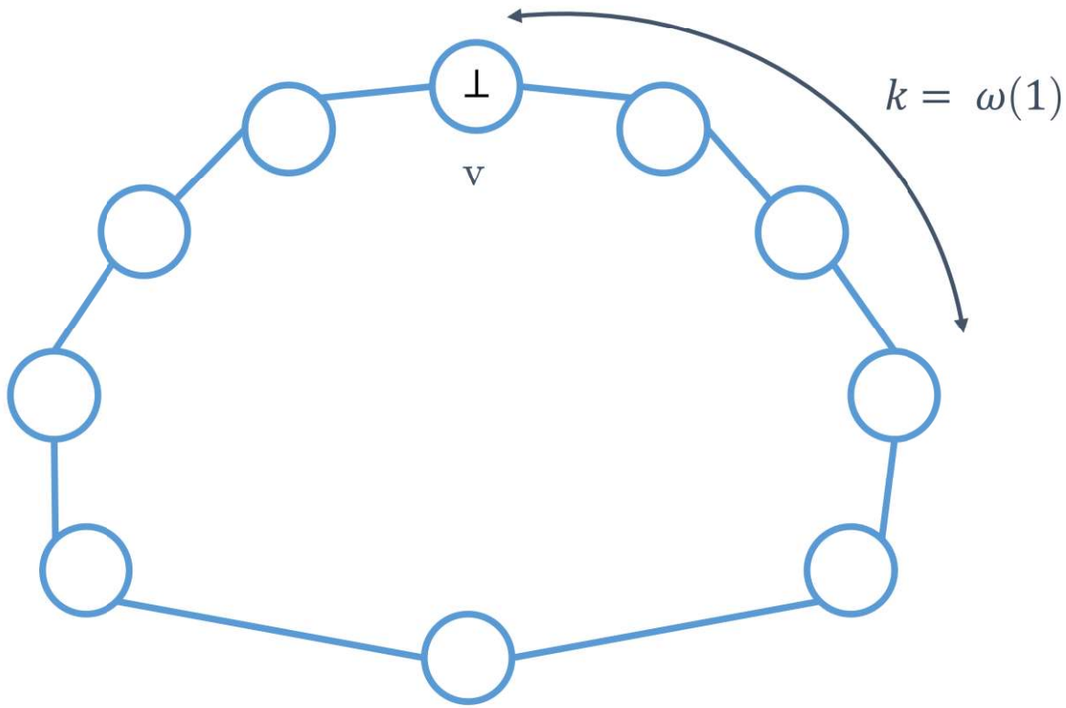
Mending in cycles

- Remove and reconnect nodes outside of the mending area

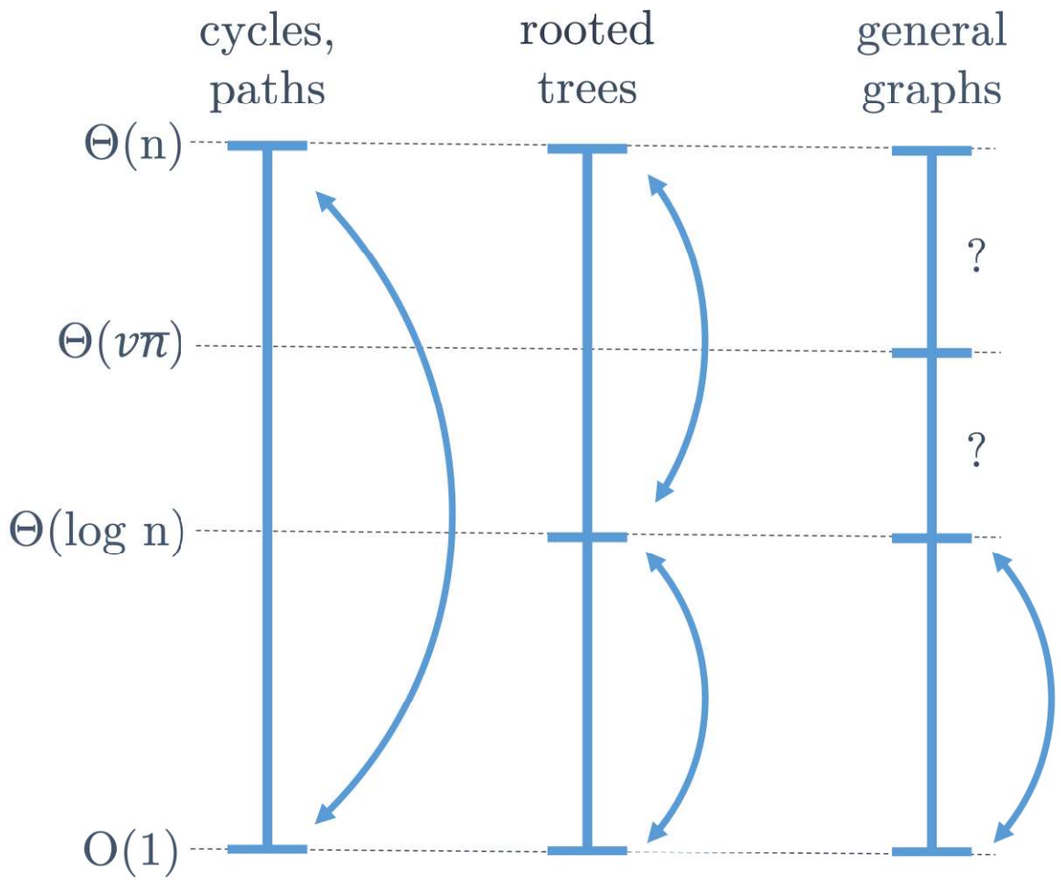


Mending in cycles

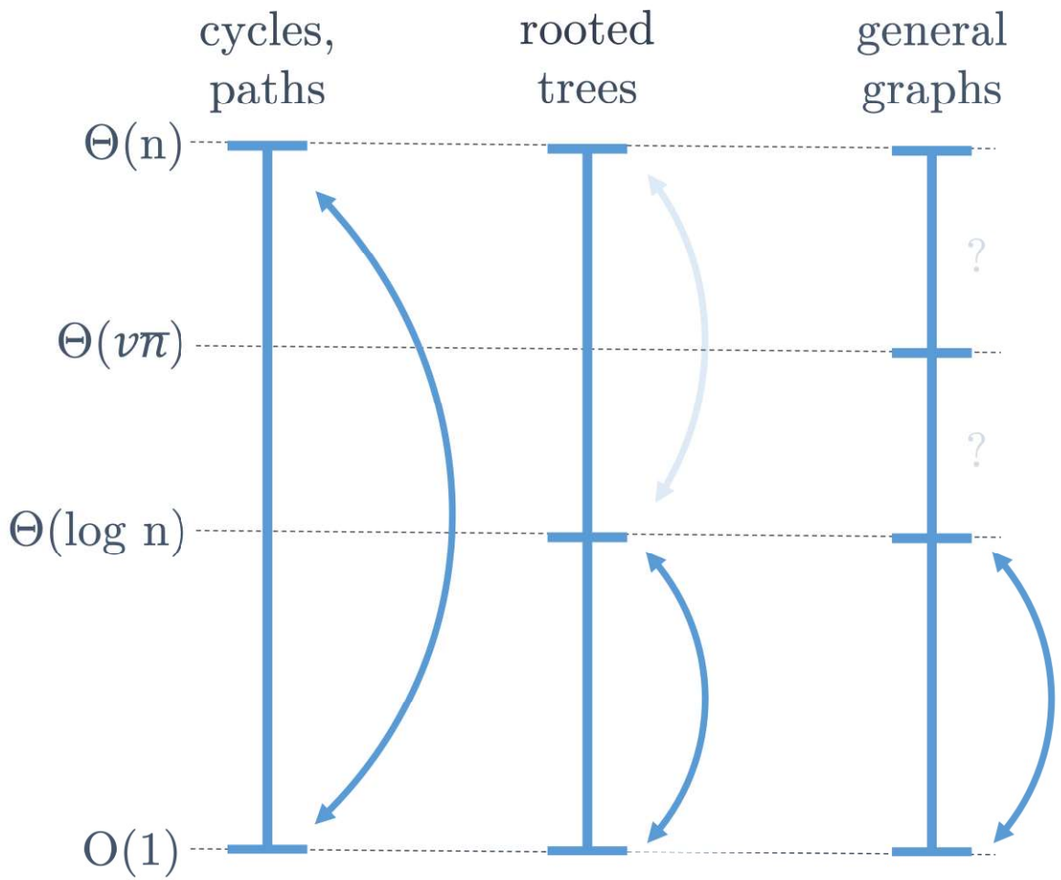
- Same problem with mending radius $\Omega(n)$



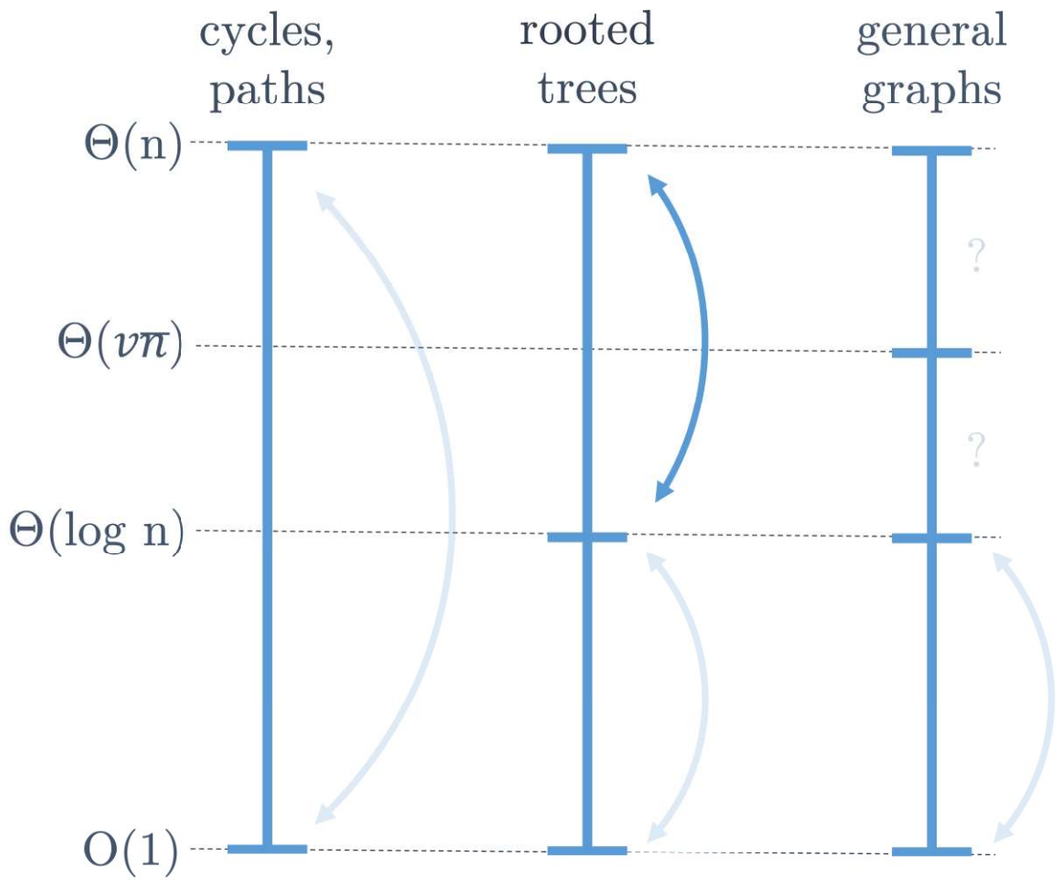
Mending LCLs



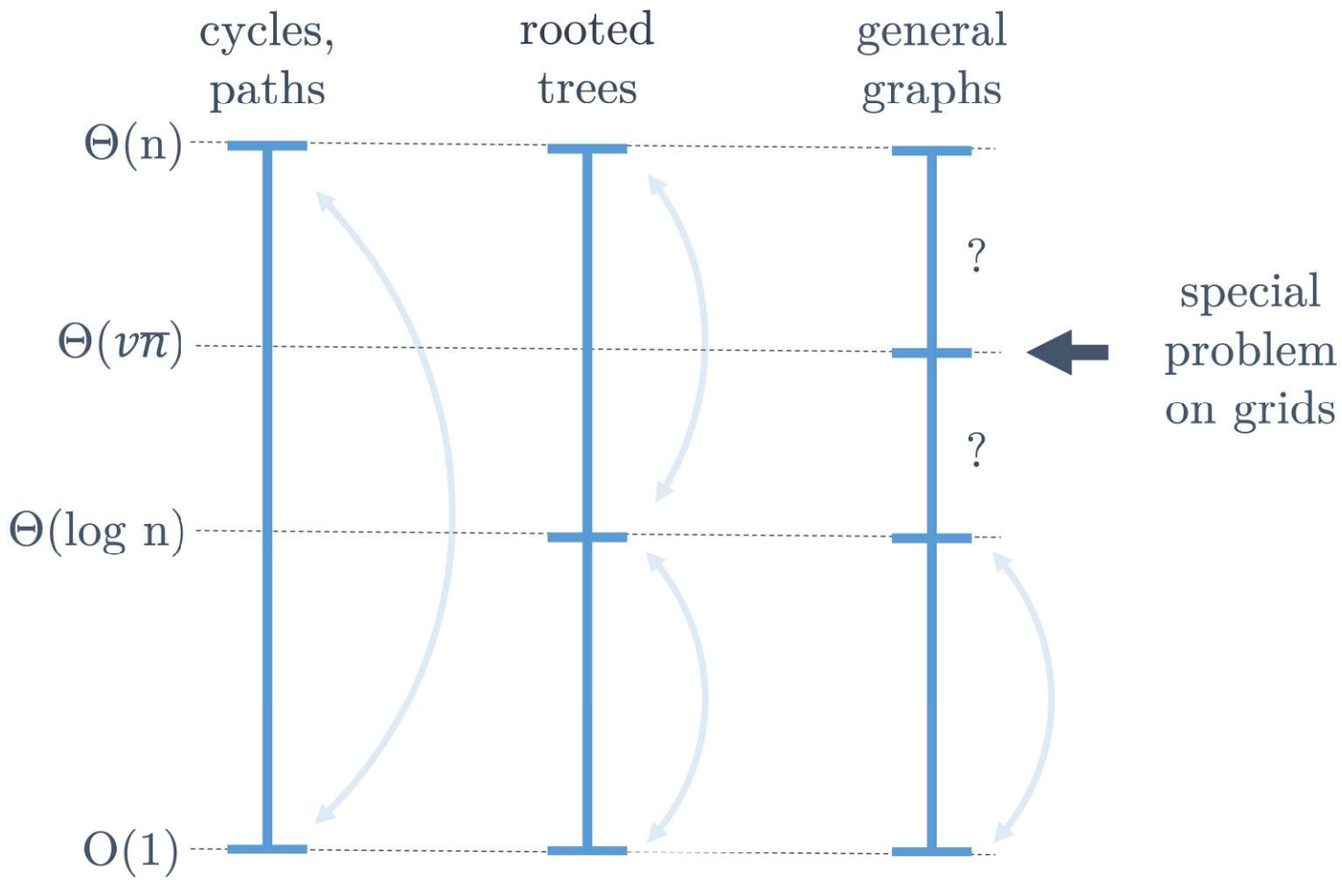
Mending LCLs



Mending LCLs

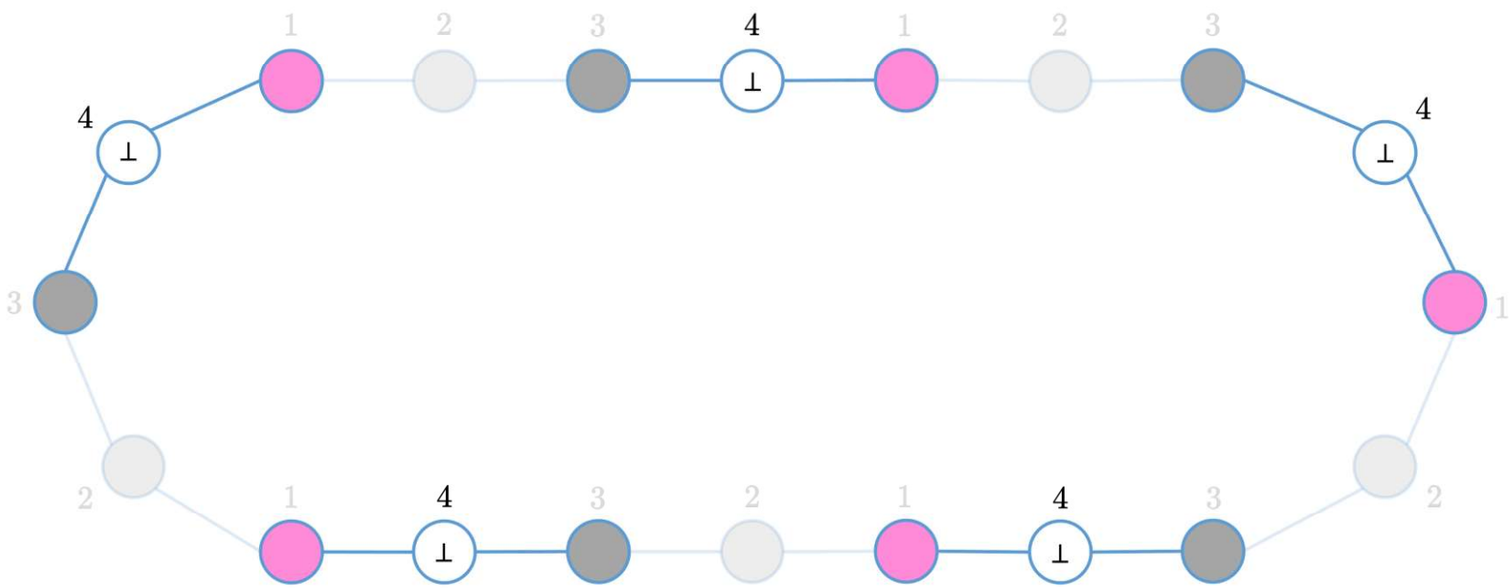


Mending LCLs



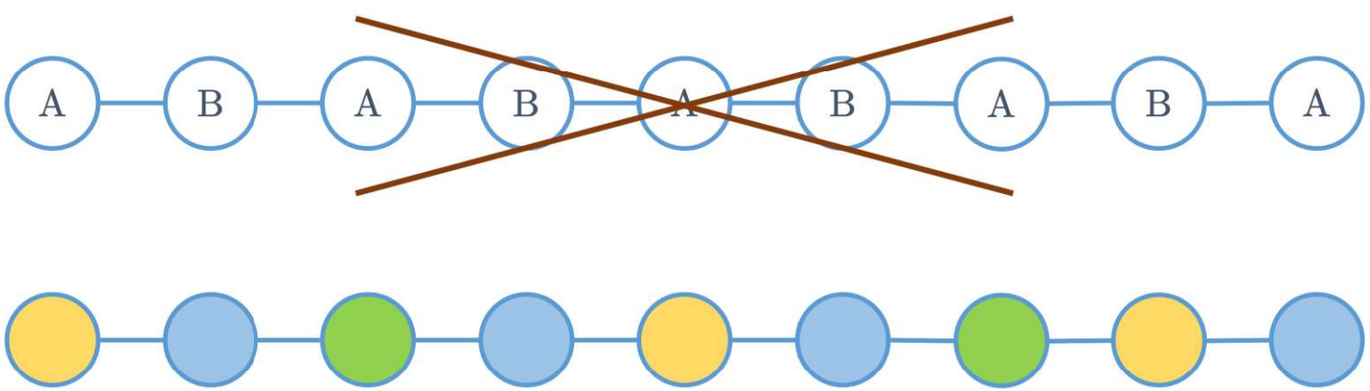
Summary

- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL



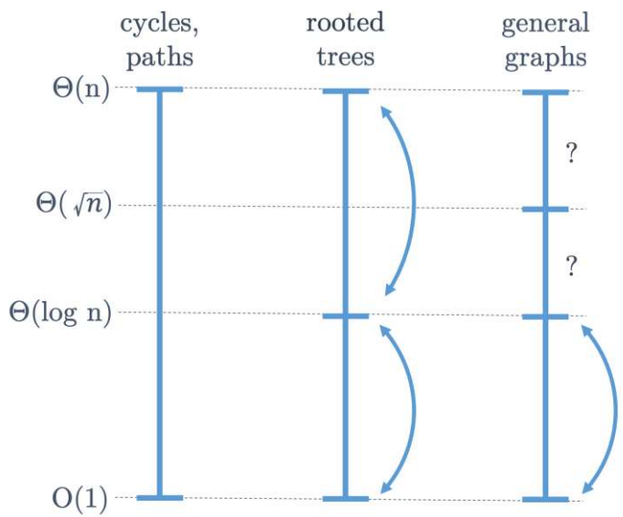
Summary

- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL
- Under restrictions, $O(\log^* n)$-solvable problems can be made radius-$O(1)$ mendable



Summary

- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL
- Under restrictions, $O(\log^* n)$-solvable problems can be made radius-$O(1)$ mendable
- Full classification of mending complexity on paths, cycles, and rooted trees



Summary

- Constant-radius mendability implies $O(\log^* n)$-solvability in LOCAL
- Under restrictions, $O(\log^* n)$-solvable problems can be made radius-$O(1)$ mendable
- Full classification of mending complexity on paths, cycles, and rooted trees