## Byzantine Agreement with Interval Validity




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Distributed Auction


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## Distributed Auction

- Fully connected graph
- Synchronous communication


Distributed Auction with Byzantine Parties


## While Byzantine parties...

- lie about inputs
- pretend to have different bids
- know all other bids
- collaborate
- know the protocol
- are unpredictable

What's the fair price?


## Valdidity Condition

- Remove too high bids



## Validity Condition

- Remove too high bids
- Remove loo low bids



## Validity Condition



Validity Condition


## Validity Condition



## Validity Condition



Interval Validity

Given t Byzantine parties

we can accept all bids which are at most $t / 2$ positions away from the wanted bid.

## Want ...

# a synchronous Byzantine agreement algorithm that satsfies 

Interval Validity

## Naive Algorithm

- Byzantine Agreement on each bid
- Can be done in $t+1$ rounds exchanging $O\left(n^{\wedge} 4\right)$ messages



## Naive Algorithm

- Choose the k-th smallest value



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- Choose the median between the $\mathbf{k}$-th smallest and the ( $\mathbf{k}+\mathbf{t}$ ) smallest value



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## Lower Bound

10 bids, 3 Byzantine, look for the 4th smallest
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- each bidder locally chooses an approximation for the $k$-th smallest value
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$t+1$ rounds and $O\left(n^{\wedge} 3\right)$ messages


## Special Cases - Minimum

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## Special Cases - Median

13 bids, 4 Byzantine, look for the median

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## Special Cases - Median

13 bids, 4 Byzantine, look for the median


## What did we achive?

Cow at an auction...

Fair price...

Bid close to the 4th smallest bid...
any real-valued agreement
any reasonable agreement value (also min, max, or median)

Interval Validity


